Diffractive Optics

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Grades	**Midterm Exam 30%, Final Exam 30%, Homework 20%, Attend 10%**
Textbook	**Element of Photonics, Volume I (Keigo Iizuka, New York, 1986)** **Introduction to Fourier Optics (Joseph Goodman, McGraw-Hill)**
Homepage	**http://optics.hanyang.ac.kr/~shsong**

Nature of diffraction

Hierarchy of the character of light

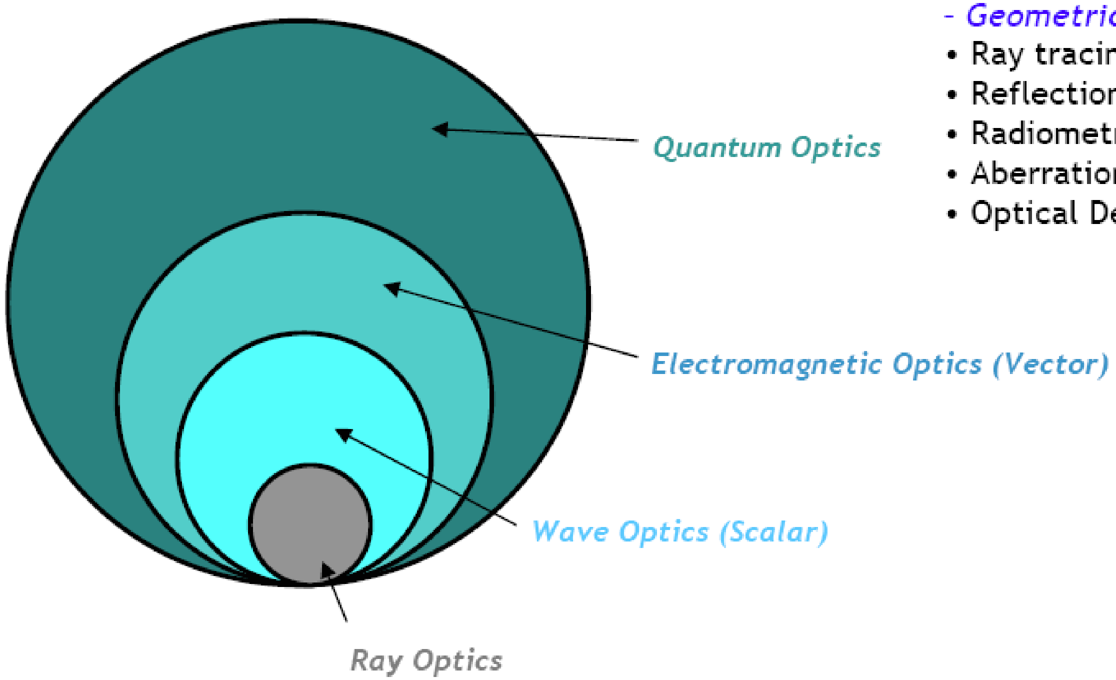
- *Quantum mechanics*
• Photon/quanta
• Probability density
• Wave function representing particles

- *Physical optics*
• Interference
• Diffraction
• Polarization

- *Geometrical optics*
• Ray tracing with wavelength short
• Reflection/refraction
• Radiometry
• Aberrations
• Optical Design

Increased Computation Speed

Increased Accuracy



Ref: Fundamental of Photonics, B.E.A. Saleh and M. C. Teich

From Grimaldi to Maxwell

- Descartes (1596-1650)
  - Considered the nature of light
  - Light was pressure transmitted through the aether
- Galileo (1564-1642)
  - Experimental methods
- Snell (1621)
  - Refraction of light at interface
- Fermat (1601-1665)
  - "Principle of Least Time"
  - Refraction laws verified
- Father Grimaldi (1618-1663)
  - First noticed "diffraction"
  - Note: diffraction is the bending of light not caused by refraction
- Newton (1642-1727)
  - Discovered basic qualities of color
  - White light could be split up into colors
  - Experiments with prisms and light and "refrangibility" or bending of light at an interface

- Huygens (1629-1695)
  - Wave propagation of light
  - Polarization of light
  - Laws of reflection and refraction
- Young (1773-1829)
  - Wave theory
  - Interference (colors of thin films)
- Fresnel (1788-1827)
  - Confirmed wave theory of propagation and diffraction
  - Influence of earth's motion of light propagation
  - Interference of polarized rays of light (light no longitudinal)
  - Reflection and polarization
  - Cause of dispersion
- Maxwell (1831-1879)
  - Theoretically unified electricity and magnetism
  - Showed possibility of electromagnetic waves propagating with velocity that could be calculated
  - Electrostatics, magnetostatics , induction, EM waves and optics unified under single theory
- Lord Rayleigh (scientific work 1899-1920)
  - Investigated waves propagation and scattering
  - Examined scattering from small particles
  - Studied wave interactions with periodic structures

Definition of diffraction

"diffractio" , **Francesco Grimaldi (1600s)**

The effect is a general characteristics of wave phenomena occurring whenever a portion of a wavefront, be it sound, a matter wave, or light, is obstructed in some way.

- Diffraction is any deviation from geometric optics that results from the obstruction of a light wave, such as sending a laser beam through an aperture to reduce the beam size. Diffraction results from the interaction of light waves with the edges of objects.
- The edges of optical images are blurred by diffraction, and this represents a fundamental limitation on the resolution of an optical imaging system.
- There is no physical difference between the phenomena of interference and diffraction, both result from the superposition of light waves. Diffraction results from the superposition of many light waves, interference results from the interference of a few light waves.

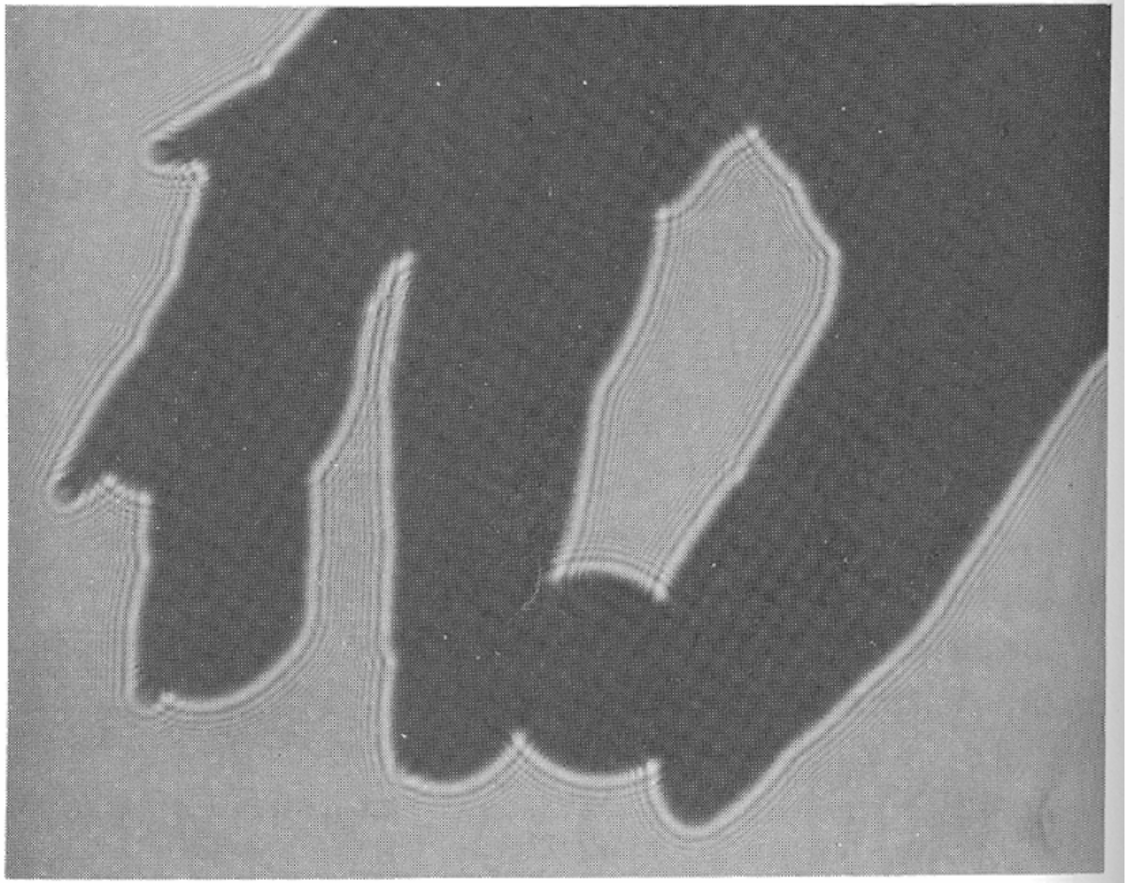
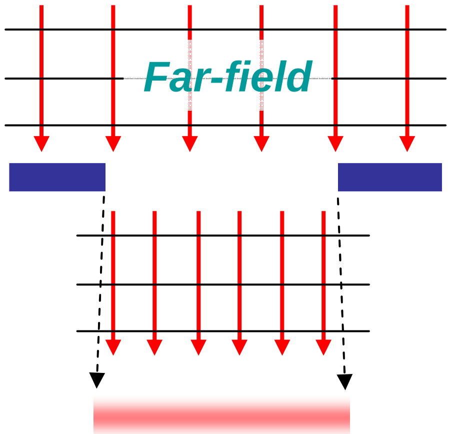


Figure 10.1 The shadow of a hand holding a dime, cast directly on 4 × 5 Polaroid A.S.A. 3000 film using a He–Ne beam and no lenses. (Photo by E.H.)

Hecht,
Optics,
Chapter 10

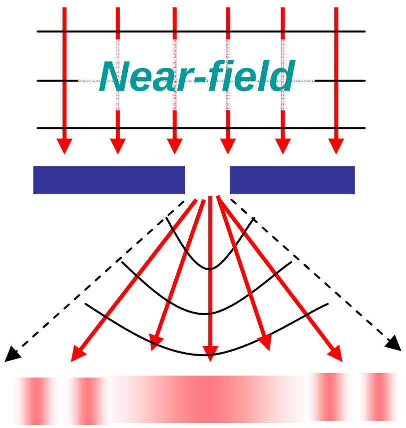
Regimes of Diffraction Optical Elements

d > λ



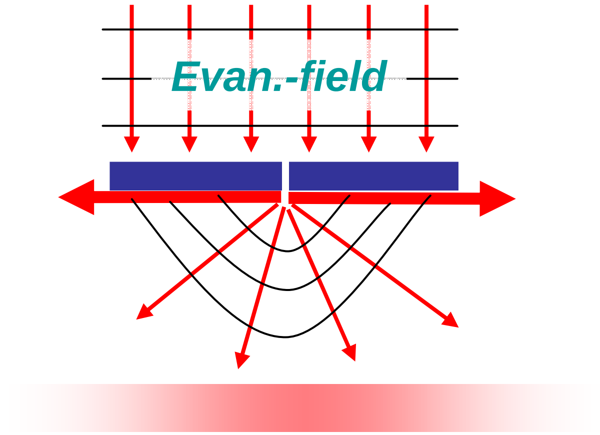
Micro lens
DOE lens
Hybrid lens
BLU
LED lighting
Beam shaping

d ~ λ



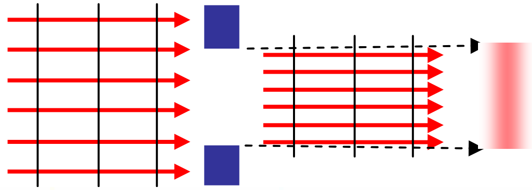
Flexible BLU
Beam shaping
LED lighting
Resonance grating
WDM filters
DFB, DBR, …
PhC device
Silicon device

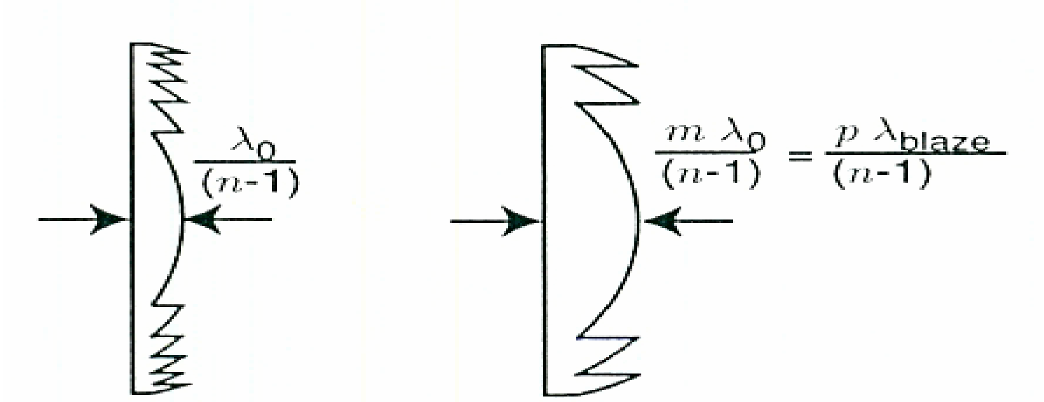
d < λ



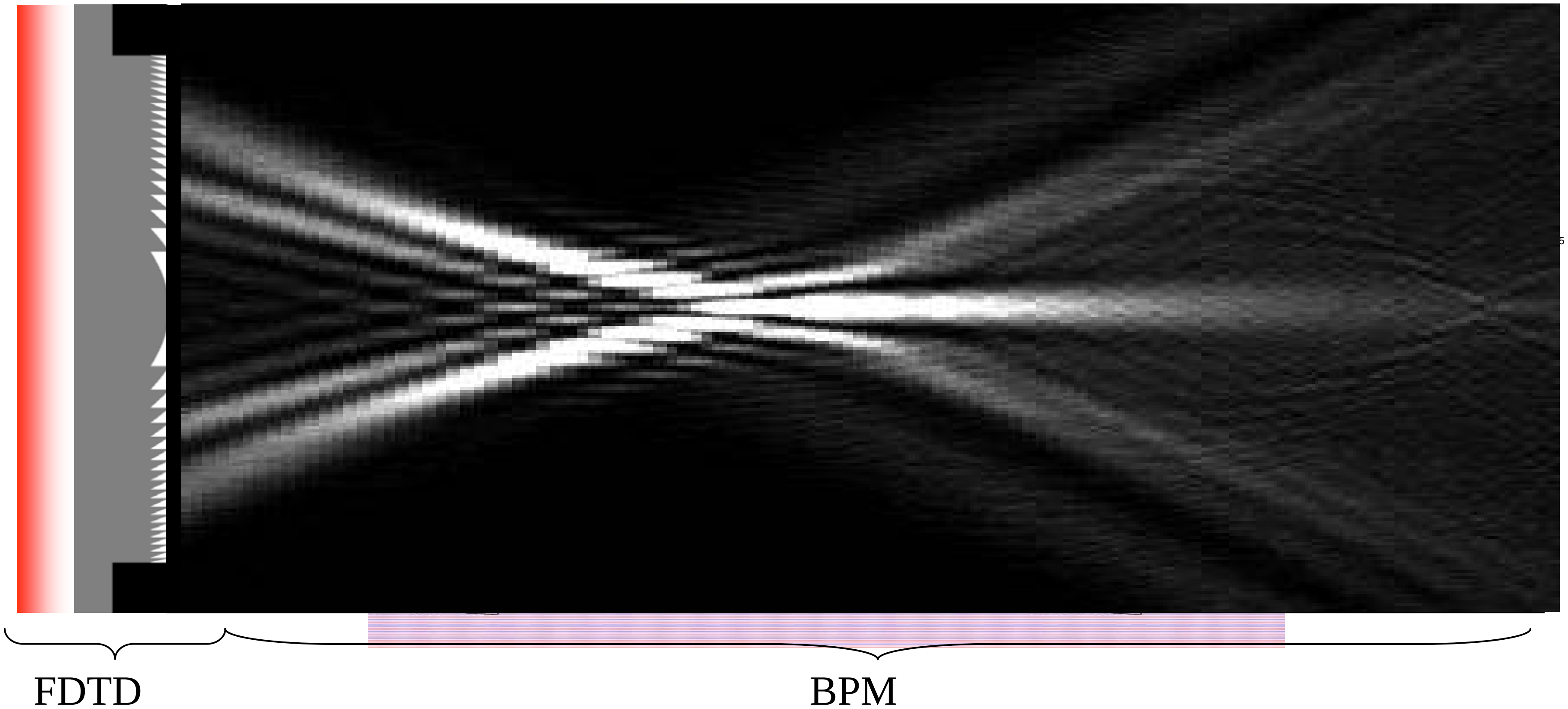
Super lens
CDEW
Metal wire
SPP waveguide
Nano-photonics

d > λ



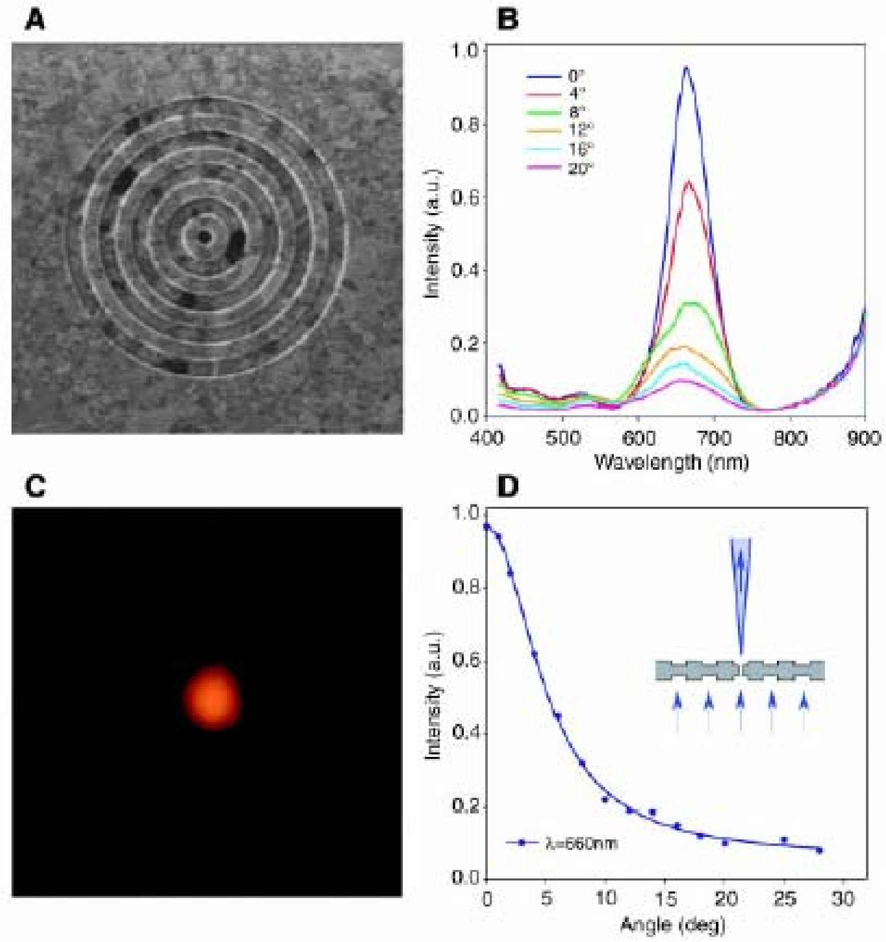


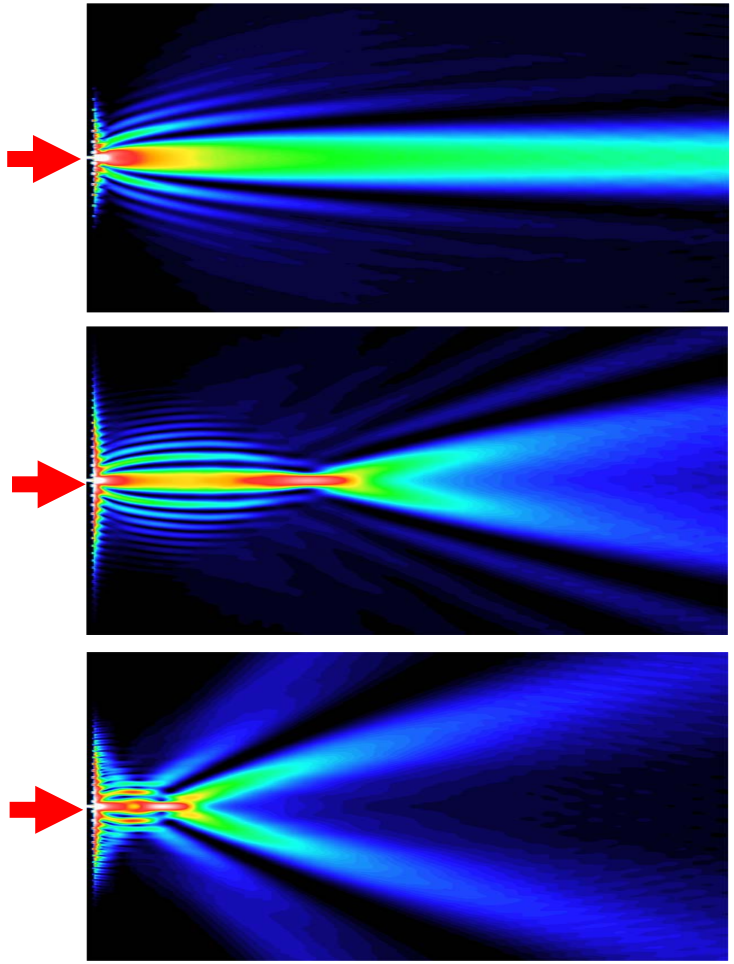
$\frac{\lambda_0}{(n-1)}$
$\frac{m\,\lambda_0}{(n-1)} = \frac{p\,\lambda_{blaze}}{(n-1)}$



FDTD
BPM

$d < \lambda$ Light transmission through a metallic subwavelength hole

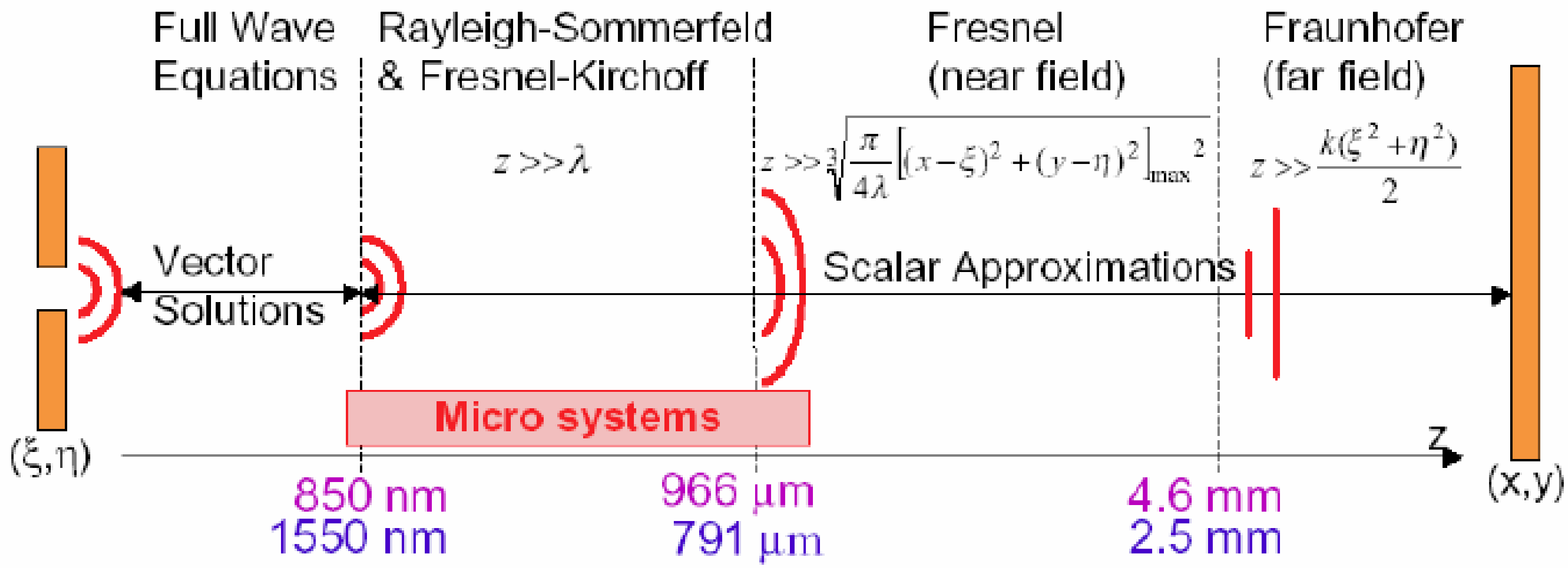




Ag film, hole diameter=250nm,
groove periodicity=500nm,
groove depth=60nm, film thickness=300nm

Science, Vol. 297, pp. 820-822, 2 August 2002.

Regimes of Diffraction



Examples: 50 µm Aperture, 200 µm Observation, λ=850 nm, λ=1550 nm

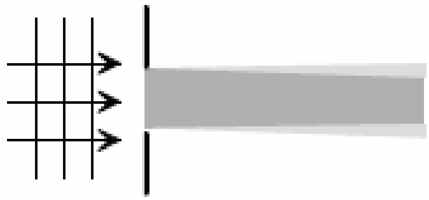
Fraunhofer Approximation - Assume planar wavefronts

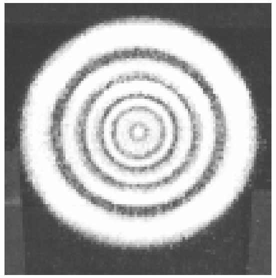
Fresnel Approximation - Assume parabolic wavefronts

Rayleigh-Sommerfeld Formulation - Spherical wavefronts

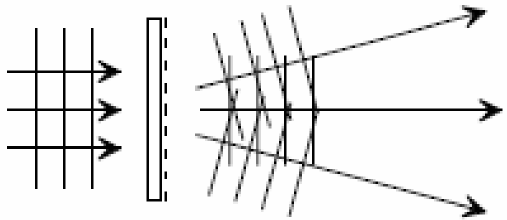
Typical diffraction phenomena

Fresnel diffraction

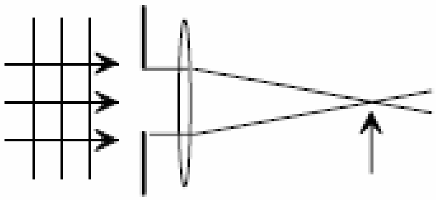


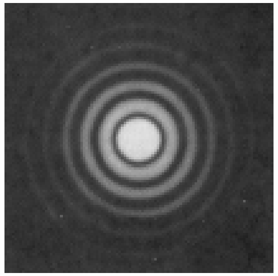


Grating: periodic structure – diffraction orders

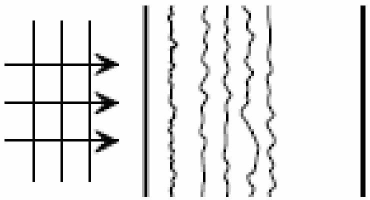


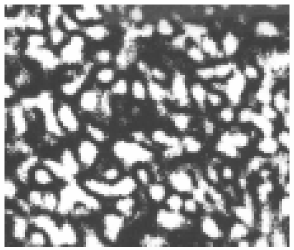
Fraunhofer diffraction - Airyn pattern





Diffraction from rough surface - speckle





History:

Grimaldi, 1665	described the phenomenon
Huygens, 1678	wave theory of light
Fresnel, 1818	intuitive explanation
Kirchhoff, 1882	mathematical formulation

Handbook of Optics, Volume I: Fundamentals, Techniques, and Design
Optical Society of America, McGraw-Hill, Inc.

Chapter 3. Diffraction *A. S. Marathay* 3.1

The electric field **E** obeys the wave equation in free space or a vacuum

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \tag{1}$$

where c is the velocity of light in a vacuum. Each cartesian component $\mathbf{E}_j$, $(j = x,\ y,\ z)$ obeys the equation and, as such, we use a scalar function $\psi(\mathbf{r}, t)$ to denote its solutions, where the radius vector $\mathbf{r}$ has components, $\mathbf{r} = \hat{i}x + \hat{j}y + \hat{k}z$.

Fourier transform on time, $\psi(\mathbf{r}, t) = \int \hat{\psi}(\mathbf{r}, \nu) \exp(-i2\pi\nu t)\, d\nu$ (2)

The spectrum $\hat{\psi}(\mathbf{r}, \nu)$ obeys the Helmholtz equation, $\nabla^2\hat{\psi} + k^2\hat{\psi} = 0$ (3)

with the propagation constant $k = 2\pi/\lambda = 2\pi\nu/c \equiv \omega/c$.

As a solution of the Helmholtz equation, a plane wave being harmonic in time as well as in space,

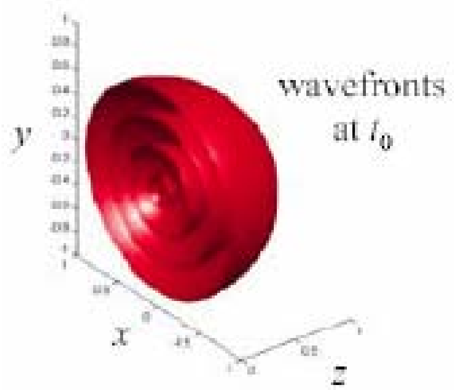
$$\psi(\mathbf{r}, t) = A \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$

For convenience of operations, a complex form frequently is used. For example,

$$\psi(\mathbf{r}, t) = A \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$

An expanding spherical wave may be written in the form,

$$\psi(r, t) = \frac{A}{r} \cos(kr - \omega t)$$



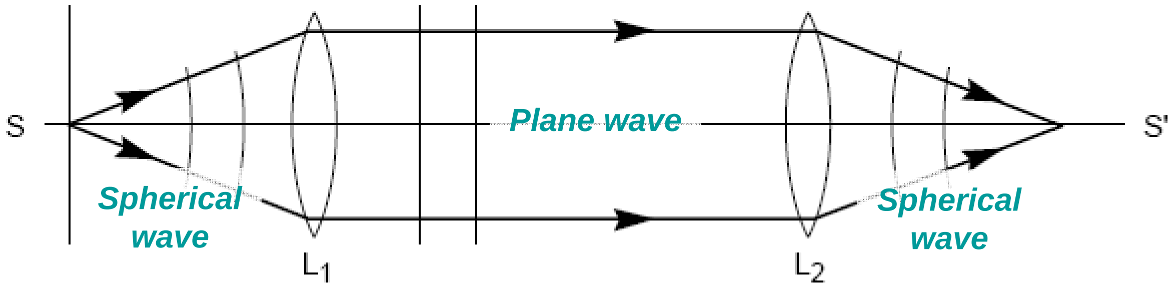


FIGURE 1 Experimental layout to describe the notation used for spherical and plane waves. S: pinhole source. L_1, L_2: lenses. S': image.

HUYGENS-FRESNEL CONSTRUCTION

Without the benefit of a fundamental theory based on Maxwell's equations and the subsequent mathematical development, Huygens sought to describe wave propagation in the days before Maxwell. Waves are characterized by constant-phase surfaces, called *wavefronts*. If the initial shape at time t of such a wavefront is known in a vacuum or in any medium, Huygens proposed a geometrical construction to obtain its shape at a later time, $t + \Delta t$. He regarded each point of the initial wavefront as the origin of a new disturbance that propagates in the form of secondary wavelets in all directions with the same speed as the speed v of propagation of the initial wave in the medium. These secondary wavelets of radii $v\,\Delta t$ are constructed at each point of the initial wavefront. A surface tangential to all these secondary wavelets, called the *envelope* of all these wavelets, is then the shape and position of the wavefront at time $t + \Delta t$. With this construct Huygens explained the phenomena of reflection and refraction of the wavefront. To explain the phenomenon of diffraction, Fresnel modified Huygens' construction by attributing the property of mutual interference to the secondary wavelets (see Chap. 2). The modified Huygens construction is called the Huygens-Fresnel construction. With further minor modifications it helps explain the phenomenon of diffraction and its various aspects, including those that are not so intuitively obvious.

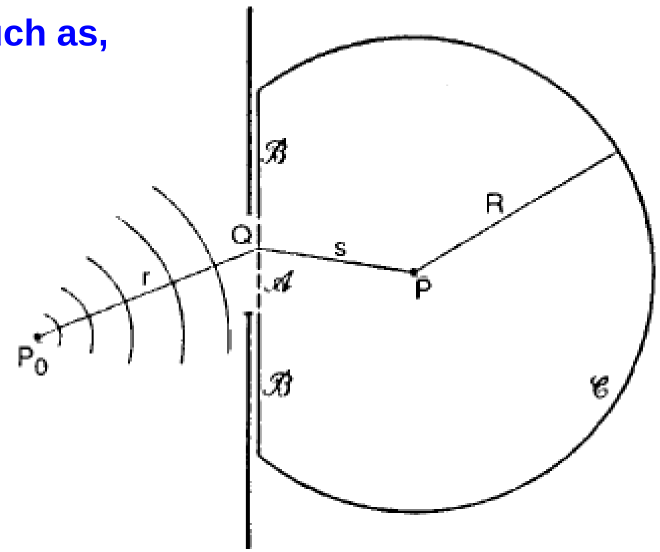
From this concept of the Huygens-Fresnel construction,
in this class we will develop some mathematical formulas, such as,

Fresnel-Kirchhoff diffraction formula

$$\psi(P) = -\left(\frac{ia}{2\lambda}\right)\iint_A \left[\frac{\exp(ikr)}{r}\right]\left[\frac{\exp(iks)}{s}\right][\cos(n,r) - \cos(n,s)]$$

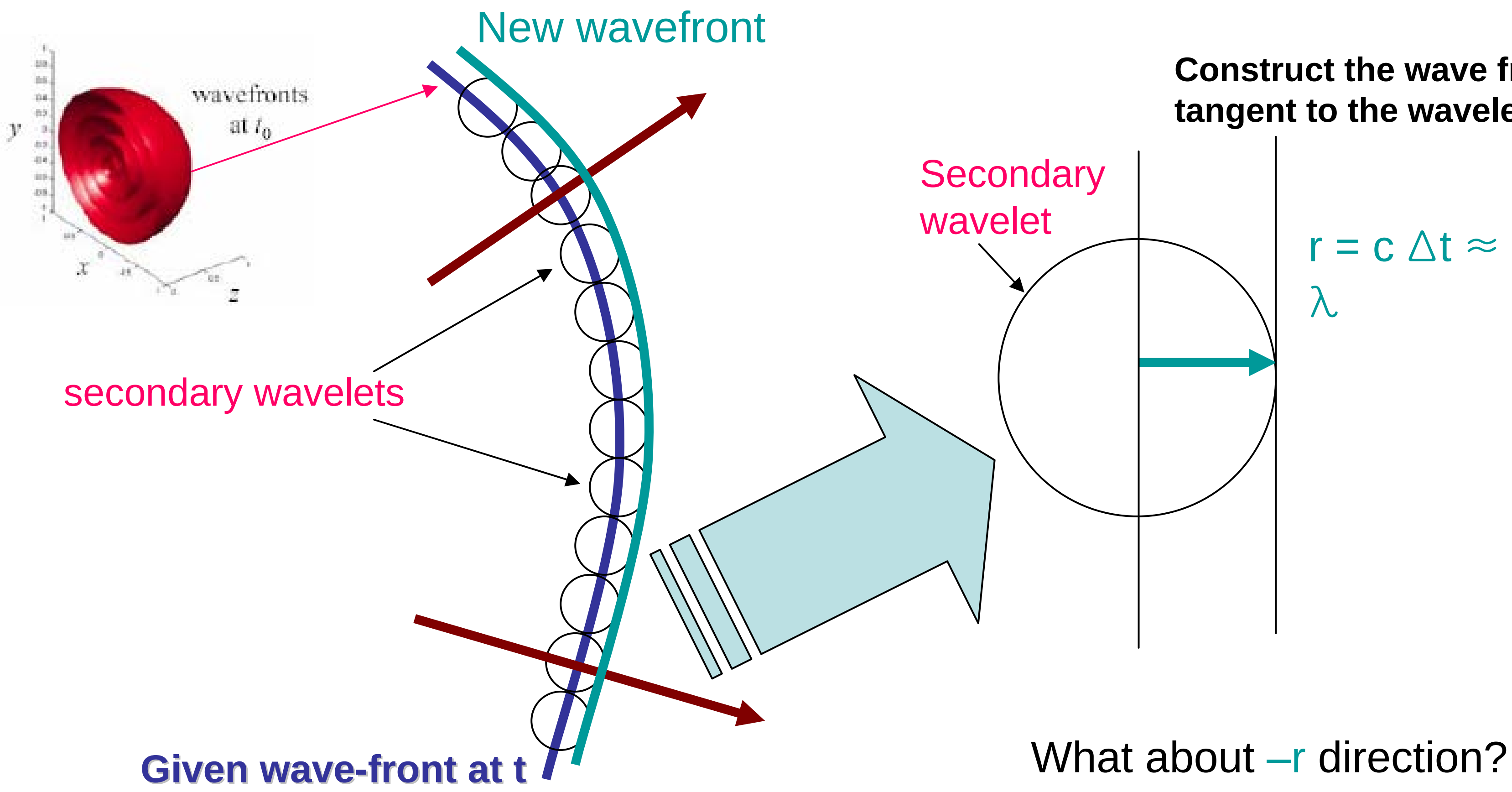
Rayleigh-Sommerfeld diffraction formula

$$\psi(P) = -\left(\frac{ia}{\lambda}\right)\iint_A \left[\frac{\exp(ikr)}{r}\right]\left[\frac{\exp(iks)}{s}\right]\cos(n,s)\,dS$$



Huygens' wave front construction

Every <u>point</u> on a wave front is a source of secondary wavelets.
i.e. particles in a medium excited by electric field (E) re-radiate in all directions
i.e. in vacuum, E, B fields associated with wave act as sources of additional fields



(π-phase delay when the secondary wavelets, Hecht, 3.5.2, 3nd Ed)

Allow wavelets to evolve for time Δt

Huygens-Fresnel principle

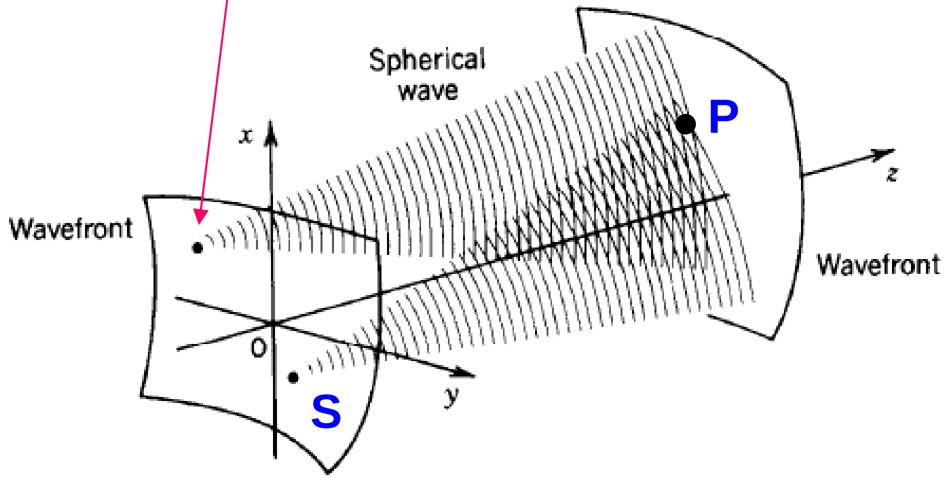
"Every unobstructed point of a wavefront, at a given instant in time, serves as a source of secondary wavelets (with the same frequency as that of the primary wave).
The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitude and relative phase)."

Huygens's principle:
By itself, it is unable to account for the details of the diffraction process.
It is indeed independent of any wavelength consideration.

Fresnel's addition of the concept of interference

HUYGENS-FRESNEL CONSTRUCTION

The Huygens–Fresnel principle. Each point on a wavefront generates a spherical wave.



The total contribution to the disturbance at P is expressed as an area integral over the primary wavefront,

$$\psi(P) = A\,\frac{\exp\left[-i(\omega t - kr_0)\right]}{r_0} \iint_S \frac{\exp(iks)}{s} K(\chi)\, dS \tag{8}$$

Huygens' Secondary wavelets on the wavefront surface S

HUYGENS-FRESNEL CONSTRUCTION : *Fresnel Zones*

The total contribution to the disturbance at P is expressed as an area integral over the primary wavefront,

$$\psi(P) = A\,\frac{\exp\left[-i(\omega t - kr_0)\right]}{r_0} \iint_S \frac{\exp(iks)}{s} K(\chi)\, dS \qquad (8)$$

Spherical wave from source P_o

Obliquity factor:
unity where χ=0 at C
zero where $\chi=\pi/2$ at high enough zone index

Huygens' Secondary wavelets on the wavefront surface S

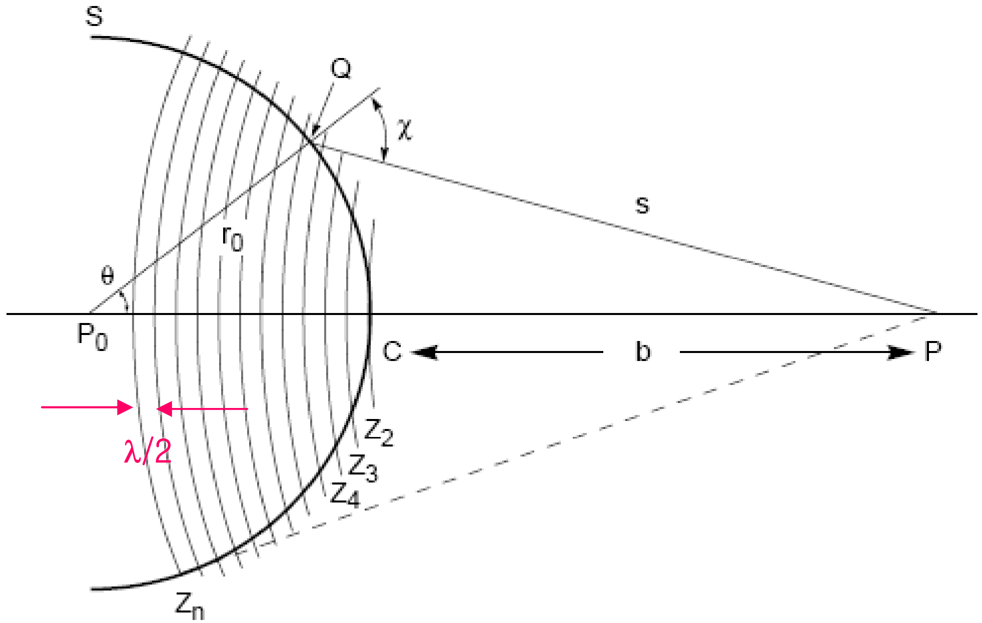
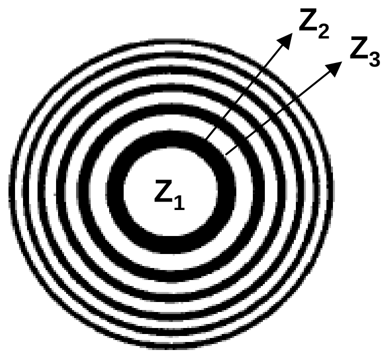


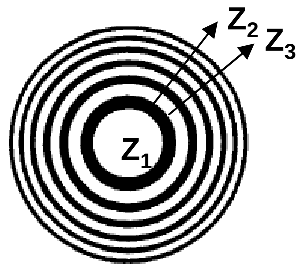
FIGURE 2 Fresnel zone construction. P_0: point source. S: wavefront. r_0: radius of the wavefront b: distance CP. s: distance QP. (*After Born and Wolf*[1].)



HUYGENS-FRESNEL CONSTRUCTION : Fresnel Zones

$$\psi(P) = A\,\frac{\exp[-i(\omega t - kr_0)]}{r_0}\iint_S \frac{\exp(iks)}{s}K(\chi)\,dS$$

The average distance of successive zones from P differs by λ/2 -> half-period zones.
Thus, the contributions of the zones to the disturbance at P alternate in sign,



$$\psi(P) = \psi_1 - \psi_2 + \psi_3 - \psi_4 + \psi_5 - \psi_6 + \ldots$$

where ψ_j stands for the contribution of the jth zone, $j = 1, 2, 3, \ldots$ The contribution of each annular zone is directly proportional to the zone area and is inversely proportional to the average distance of the zone to the point of observation P. The ratio of the zone area to its average distance from P is independent of the zone index j. Thus, in summing the contributions of the zones we are left with only the variation of the obliquity factor, $K(\chi)$. To a good approximation, the obliquity factors for any two adjacent zones are nearly equal and for a large enough zone index j the obliquity factor becomes negligible. The total disturbance at the point of observation P may be approximated by

$$\psi(P) = 1/2(\psi_1 \pm \psi_n)$$

(1/2 means averaging of the possible values, more details are in 10-3, Optics, Hecht, 2nd Ed)

For an unobstructed wave, the last term $\psi_n = 0$.

$$\psi(P) = 1/2\psi_1$$

$$= \frac{A}{r_0 + b}\lambda \exp\{-i[\omega t - k(r_0 + b) - \pi/2]\}$$

Whereas, a freely propagating spherical wave from the source P_o to P is

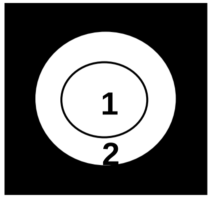
$$\psi(P) = \frac{A}{r_0 + b}\exp\{-i[\omega t - k(r_0 + b)]\}$$

Therefore, one can assume that the complex amplitude of $\exp(iks)/s$

$$[1/\lambda \exp(-i\pi/2)]\ \exp(iks)/s$$

$$= \frac{1}{i\lambda}\left(\frac{\exp(iks)}{s}\right)$$

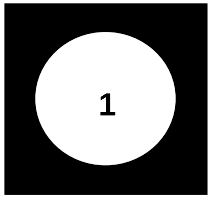
HUYGENS-FRESNEL CONSTRUCTION : *Diffraction of light from circular apertures and disks*



(a) The first two zones are uncovered,

$\psi(P) = \psi_1 - \psi_2 = 0!$ (consider the point P at the on-axis P)

since these two contributions are nearly equal,



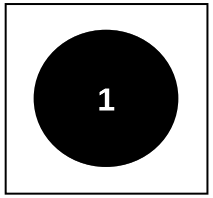
(b) The first zone is uncovered if point P is placed father away,

$\psi(P) = \psi_1$

which is twice what it was for the unobstructed wave!

: Babinet principle

$$\psi_S(P) + \psi_{CS}(P) = \psi_{UN}(P)$$



(c) Only the first zone is covered by an opaque disk,

$$\psi(P) = -\psi_2 + \psi_3 - \psi_4 + \psi_5 - \psi_6 + \cdots = -\tfrac{1}{2}\psi_2$$

which is the same as the amplitude of the unobstructed wave.

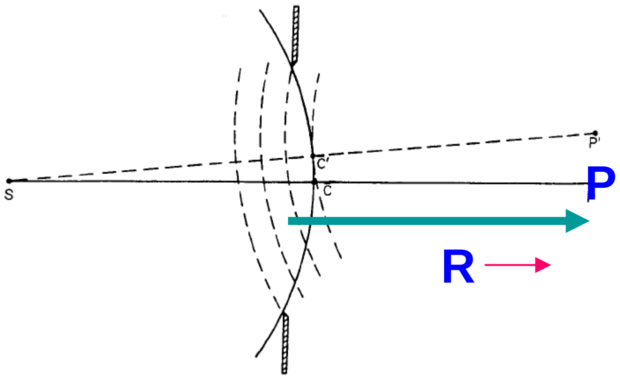


FIGURE 4 The redrawn zone structure for use with an off-axis point P'. (*After Andrews.*[8])

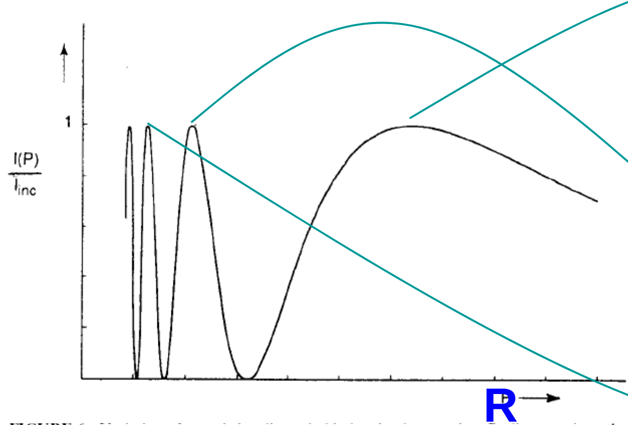
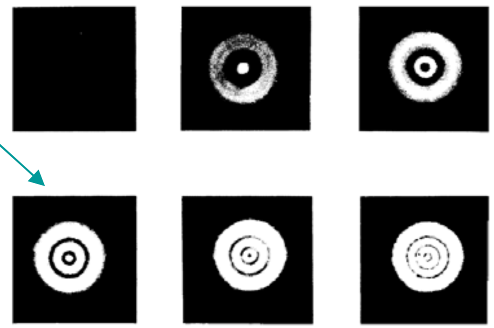


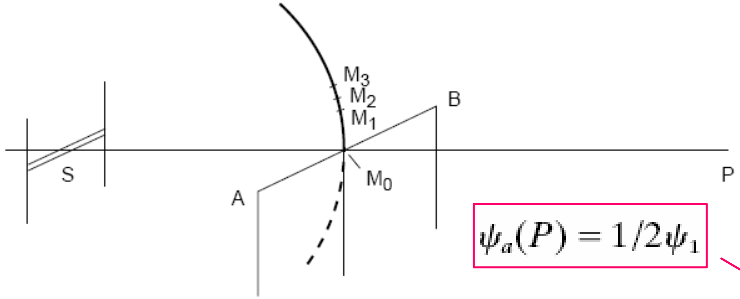
FIGURE 6 ... ng the axis behind

Variation of on-axis irradiance



Diffraction patterns from circular apertures

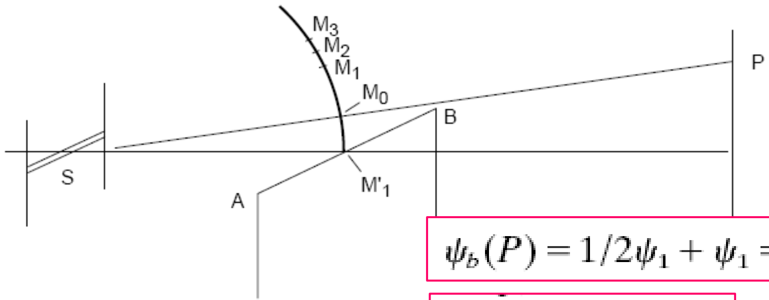
HUYGENS-FRESNEL CONSTRUCTION : Straight edge



$$\psi_a(P) = 1/2\psi_1$$

(a)

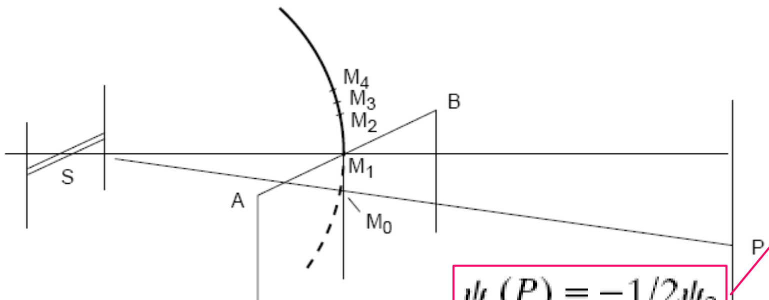
At the edge



$$\psi_b(P) = 1/2\psi_1 + \psi_1 = 3/2\psi_1$$

(b)

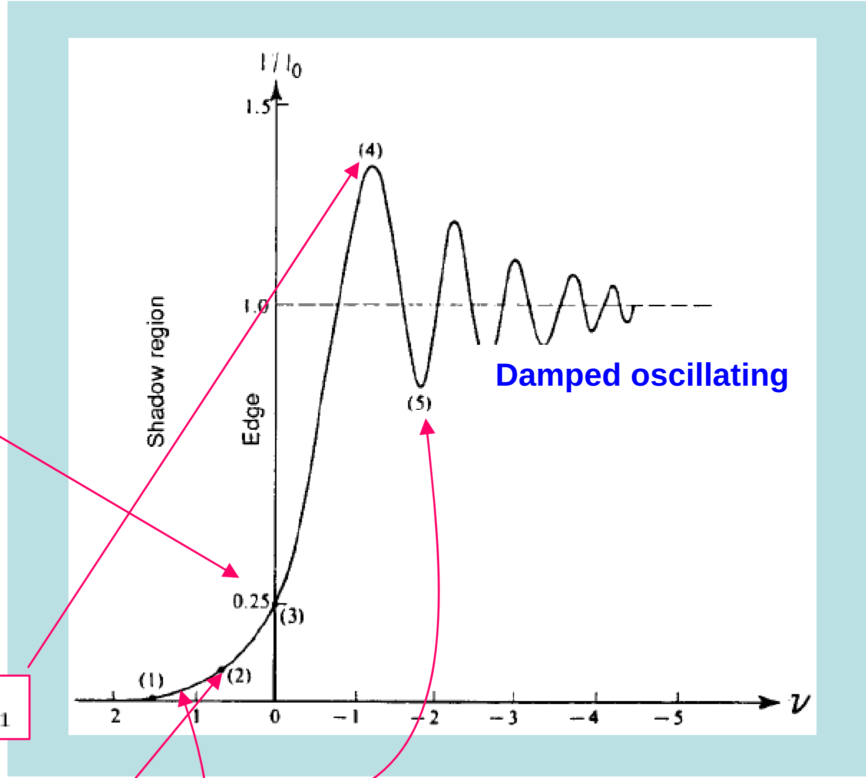
$$\psi(P) = 3/2\psi_1 - \psi_2$$



$$\psi_c(P) = -1/2\psi_2$$

(c)

$$\psi(P) = 1/2\psi_3$$



Damped oscillating

Monotonically decreasing

After the Huygens-Fresnel principle

Fresnel's shortcomings :

He did not mention the existence of backward secondary wavelets, however, there also would be a reverse wave traveling back toward the source. He introduce a quantity of the obliquity factor, but he did little more than conjecture about this kind.

$$\psi(P) = A\,\frac{\exp\left[-i(\omega t - kr_0)\right]}{r_0}\iint\limits_S \frac{\exp(iks)}{s}K(\chi)\,dS$$

Gustav Kirchhoff : Fresnel-Kirhhoff diffraction theory

A more rigorous theory based directly on the solution of the differential wave equation. He, although a contemporary of Maxwell, employed the older elastic-solid theory of light. He found K(χ) = (1 + cosθ)/2. K(0) = 1 in the forward direction, K(π) = 0 with the back wave.

$$\psi(P) = -\left(\frac{ia}{2\lambda}\right)\iint\limits_A \left[\frac{\exp(ikr)}{r}\right]\left[\frac{\exp(iks)}{s}\right]\left[\cos(n,r) - \cos(n,s)\right]dS$$

Arnold Johannes Wilhelm Sommerfeld : Rayleigh-Sommerfeld diffraction theory

A very rigorous solution of partial differential wave equation. The first solution utilizing the electromagnetic theory of light.

$$\psi(P) = -\left(\frac{ia}{\lambda}\right)\iint\limits_A \left[\frac{\exp(ikr)}{r}\right]\left[\frac{\exp(iks)}{s}\right]\cos(n,s)\,dS$$

Kirchhoff's theorem

Starting point: *Field known on a closed surface S.*
What is the field in a point P_0 inside S?

- Scalar approximation (polarization effects ignored)

Gauss: $\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_V \nabla\cdot\mathbf{A}\,dv$

If $\mathbf{A} = G\nabla U - U\nabla G$ (U(**r**) and G(**r**) arbitrary scalar functions)

Green II: $\oint_S (G\nabla U - U\nabla G)\cdot d\mathbf{S} = \int_V \left(G\nabla^2 U - U\nabla^2 G\right) dv$

Assume now that U and G satisfy the homogeneous wave equation and that their time dependence is $\sim e^{-i\omega t}$

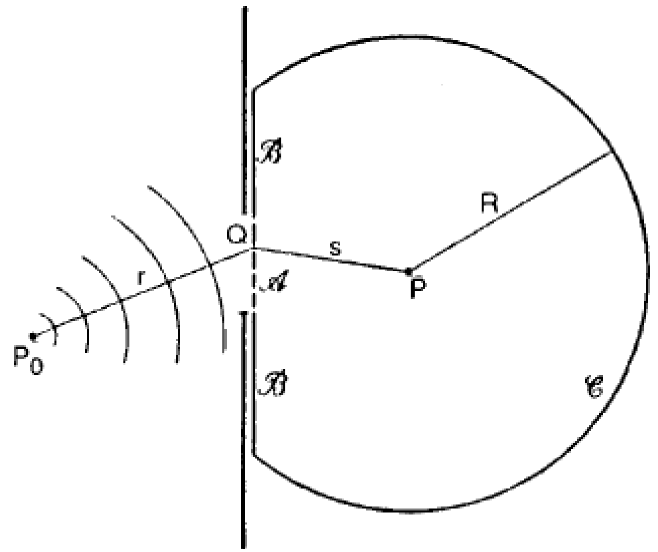
$$\left.\begin{aligned}\nabla^2 U &= \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = -k^2 U\\ \nabla^2 G &= \frac{1}{c^2}\frac{\partial^2 G}{\partial t^2} = -k^2 G\end{aligned}\right\} \qquad k^2 = \frac{\omega^2}{c^2} \qquad \Rightarrow \int_V \left(G\nabla^2 U - U\nabla^2 G\right) dv = 0$$

Fresnel-Kirchhoff diffraction formula

$$\psi(P) = -\left(\frac{ia}{2\lambda}\right)\iint_A \left[\frac{\exp(ikr)}{r}\right]\left[\frac{\exp(iks)}{s}\right][\cos(n,r) - \cos(n,s)]\,dS$$

Rayleigh-Sommerfeld diffraction formula

$$\psi(P) = -\left(\frac{ia}{\lambda}\right)\iint_A \left[\frac{\exp(ikr)}{r}\right]\left[\frac{\exp(iks)}{s}\right]\cos(n,s)\,dS$$



Fresnel diffraction

Assume: $z >> x_1, y_1; x_0, y_0$

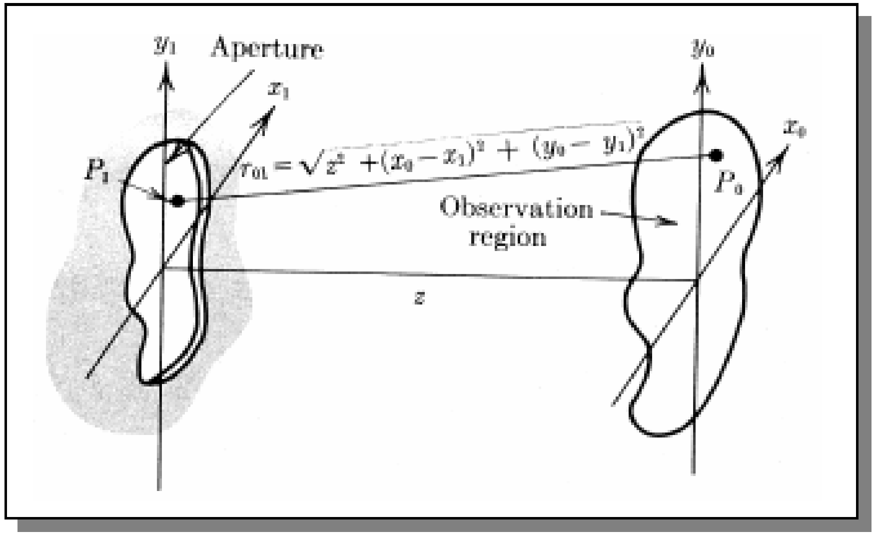
$$\iint_{\Sigma} \rightarrow \iint_{-\infty}^{\infty} \qquad \text{(U = 0 outside the aperture)}$$

Fresnel's approximation:

In the exponent:

$$\boxed{\begin{aligned} r &= \sqrt{z^2 + (x_0 - x_1)^2 + (y_0 - y_1)^2} \\ &\approx z\left[1 + \frac{1}{2}\left(\frac{x_0 - x_1}{z}\right)^2 + \frac{1}{2}\left(\frac{y_0 - y_1}{z}\right)^2\right] \end{aligned}} \qquad \left(\sqrt{1+x} \approx 1 + \frac{1}{2}x\right)$$

In the denominator: $r \rightarrow z$



$$\boxed{U(x_0, y_0) = -\frac{ik}{2\pi}\frac{e^{ikz}}{z}\iint_{-\infty}^{\infty} U(x_1, y_1)\, e^{\frac{ik}{2z}\left[(x_0 - x_1)^2 + (y_0 - y_1)^2\right]}\, dx_1 dy_1}$$

Fraunhofer diffraction

Fraunhofer's approximation:

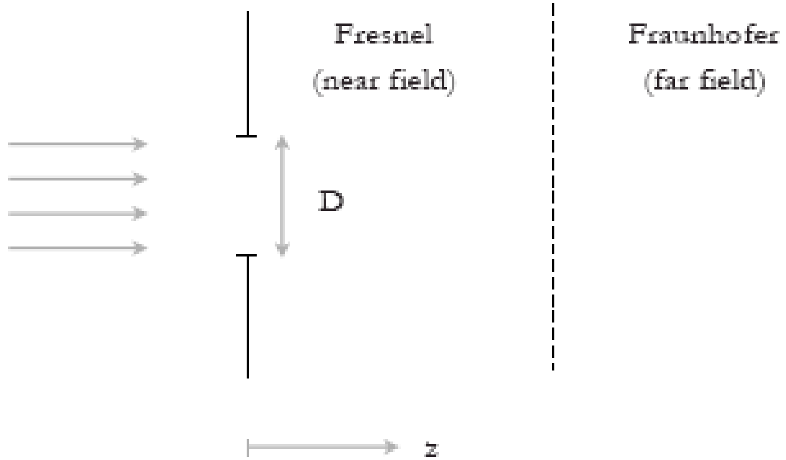
$$\frac{ik}{2z}\left[(x_0-x_1)^2+(y_0-y_1)^2\right]=\frac{ik}{2z}\left[(x_0^2+y_0^2)+\cancel{(x_1^2+y_1^2)}-2(x_0x_1+y_0y_1)\right] \qquad \text{if} \quad \boxed{\frac{k(x_1^2+y_1^2)}{2} << z}$$

$$U(x_0,y_0)=A\iint_{-\infty}^{\infty}U(x_1,y_1)\,e^{\frac{-ik}{z}(x_0x_1+y_0y_1)}\,dx_1dy_1$$

$$A=-\frac{ik}{2\pi}\frac{e^{ikz}}{z}e^{\frac{ik}{2z}(x_0^2+y_0^2)}$$

$z \sim D^2/\lambda$

D ~ 1 cm
λ ~ 1 µm
z ~100 m



$U(x_0,y_0)$ is given by the Fourier transform of $U(x_1,y_1)$

Fourier optics

Examples of Fourier transforms