Diffractive Optics

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Grades
Midterm Exam 30%, Final Exam 30%, Homework 20%, Attend 10%

Textbook
Element of Photonics, Volume I (Keigo Iizuka, New York, 1986)
Introduction to Fourier Optics (Joseph Goodman, McGraw-Hill)

Homepage
http://optics.hanyang.ac.kr/~shsong
Nature of diffraction

Hierarchy of the character of light

- Quantum mechanics
  - Photon/quanta
  - Probability density
  - Wave function representing particles

- Physical optics
  - Interference
  - Diffraction
  - Polarization

- Geometrical optics
  - Ray tracing with wavelength short
  - Reflection/refraction
  - Radiometry
  - Aberrations
  - Optical Design

Ref: Fundamental of Photonics, B.E.A. Saleh and M. C. Teich
From Grimaldi to Maxwell

- Descartes (1596-1650)
  - Considered the nature of light
  - Light was pressure transmitted through the aether
- Galileo (1564-1642)
  - Experimental methods
- Snell (1621)
  - Refraction of light at interface
- Fermat (1601-1665)
  - “Principle of Least Time”
  - Refraction laws verified
- Father Grimaldi (1618-1663)
  - First noticed “diffraction”
  - Note: diffraction is the bending of light not caused by refraction
- Newton (1642-1727)
  - Discovered basic qualities of color
  - White light could be split up into colors
  - Experiments with prisms and light and “refrangibility” or bending of light at an interface
- Huygens (1629-1695)
  - Wave propagation of light
  - Polarization of light
  - Laws of reflection and refraction
- Young (1773-1829)
  - Wave theory
  - Interference (colors of thin films)
- Fresnel (1788-1827)
  - Confirmed wave theory of propagation and diffraction
  - Influence of earth’s motion of light propagation
  - Interference of polarized rays of light (light no longitudinal)
  - Reflection and polarization
  - Cause of dispersion
- Maxwell (1831-1879)
  - Theoretically unified electricity and magnetism
  - Showed possibility of electromagnetic waves propagating with velocity that could be calculated
  - Electrostatics, magnetostatics, induction, EM waves and optics unified under single theory
- Lord Rayleigh (scientific work 1899-1920)
  - Investigated waves propagation and scattering
  - Examined scattering from small particles
  - Studied wave interactions with periodic structures
Definition of diffraction

“diffractio”, Francesco Grimaldi (1600s)

The effect is a general characteristics of wave phenomena occurring whenever a portion of a wavefront, be it sound, a matter wave, or light, is obstructed in some way.

- Diffraction is any deviation from geometric optics that results from the obstruction of a light wave, such as sending a laser beam through an aperture to reduce the beam size. Diffraction results from the interaction of light waves with the edges of objects.

- The edges of optical images are blurred by diffraction, and this represents a fundamental limitation on the resolution of an optical imaging system.

- There is no physical difference between the phenomena of interference and diffraction, both result from the superposition of light waves. Diffraction results from the superposition of many light waves, interference results from the interference of a few light waves.
Figure 10.1  The shadow of a hand holding a dime, cast directly on 4 x 5 Polaroid A.S.A. 3000 film using a He–Ne beam and no lenses. (Photo by E.H.)
Regimes of Diffraction Optical Elements

$d > \lambda$

- Micro lens
- DOE lens
- Hybrid lens
- BLU
- LED lighting
- Beam shaping

$d \sim \lambda$

- Flexible BLU
- Beam shaping
- LED lighting
- Resonance grating
- WDM filters
- DFB, DBR, ...
- PhC device
- Silicon device

$d < \lambda$

- Super lens
- CDEW
- Metal wire
- SPP waveguide
- Nano-photonics
\[ d > \lambda \]

\[ \frac{\lambda_0}{(n-1)} \]

\[ \frac{m \lambda_0}{(n-1)} = \frac{p \lambda_{\text{blaze}}}{(n-1)} \]
\[ d < \lambda \]

Light transmission through a metallic subwavelength hole

Ag film, hole diameter=250nm, groove periodicity=500nm, groove depth=60nm, film thickness=300nm

Regimes of Diffraction

Full Wave Equations

Rayleigh-Sommerfeld & Fresnel-Kirchoff

Fresnel (near field)

Fraunhofer (far field)

Examples: 50 μm Aperture, 200 μm Observation, λ=850 nm, λ=1550 nm

Fraunhofer Approximation - Assume planar wavefronts

Fresnel Approximation - Assume parabolic wavefronts

Rayleigh-Sommerfeld Formulation - Spherical wavefronts
Typical diffraction phenomena

Fresnel diffraction

Grating: periodic structure – diffraction orders

Fraunhofer diffraction - Airyn pattern

History:

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grimaldi, 1665</td>
<td>described the phenomenon</td>
</tr>
<tr>
<td>Huygens, 1678</td>
<td>wave theory of light</td>
</tr>
<tr>
<td>Fresnel, 1818</td>
<td>intuitive explanation</td>
</tr>
<tr>
<td>Kirchhoff, 1882</td>
<td>mathematical formulation</td>
</tr>
</tbody>
</table>
The electric field $E$ obeys the wave equation in free space or a vacuum

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \quad (1)$$

where $c$ is the velocity of light in a vacuum. Each cartesian component $E_j$ ($j = x, y, z$) obeys the equation and, as such, we use a scalar function $\psi(r, t)$ to denote its solutions, where the radius vector $r$ has components, $r = i x + j y + k z$.

Fourier transform on time,

$$\psi(r, t) = \int \tilde{\psi}(r, \nu) \exp(-i 2\pi \nu t) \, d\nu \quad (2)$$

The spectrum $\tilde{\psi}(r, \nu)$ obeys the Helmholtz equation,

$$\nabla^2 \tilde{\psi} + k^2 \tilde{\psi} = 0 \quad (3)$$

with the propagation constant $k = 2\pi/\lambda = 2\pi\nu/c = \omega/c$. 

**Chapter 3. Diffraction  A. S. Marathay**

3.1. Glossary / 3.1
3.2. Introduction / 3.1
3.3. Light Waves / 3.2
3.4. Huygens-Fresnel Construction / 3.4
3.5. Cylindrical Wavefront / 3.13
3.7. Vector Diffraction / 3.27
3.8. References / 3.30
As a solution of the Helmholtz equation, a plane wave being harmonic in time as well as in space,

\[ \psi(r, t) = A \cos(k \cdot r - \omega t) \]

For convenience of operations, a complex form frequently is used. For example,

\[ \psi(r, t) = A \exp[i(k \cdot r - \omega t)] \]

An expanding spherical wave may be written in the form,

\[ \psi(r, t) = \frac{A}{r} \cos(kr - \omega t) \]

**FIGURE 1** Experimental layout to describe the notation used for spherical and plane waves.  
From this concept of the Huygens-Fresnel construction, in this class we will develop some mathematical formulas, such as,

Fresnel-Kirchhoff diffraction formula

\[ \psi(P) = - \left( \frac{ia}{2\lambda} \right) \int \left[ \frac{\exp(ikr)}{r} \right] \left[ \frac{\exp(iks)}{s} \right] \left[ \cos(n,r) - \cos(n,s) \right] \]

Rayleigh-Sommerfeld diffraction formula

\[ \psi(P) = - \left( \frac{ia}{\lambda} \right) \int \left[ \frac{\exp(ikr)}{r} \right] \left[ \frac{\exp(iks)}{s} \right] \cos(n,s) \, dS \]
Huygens’ wave front construction

Every **point** on a wave front is a source of secondary wavelets. 
i.e. *particles in a medium excited by electric field (E) re-radiate in all directions*
i.e. *in vacuum, E, B fields associated with wave act as sources of additional fields*

Given wave-front at \( t \)

Allow wavelets to evolve for time \( \Delta t \)

New wavefront

Construct the wave front tangent to the wavelets

\[ r = c \Delta t \approx \lambda \]

Secondary wavelet

What about \(-r\) direction? 
(\(\pi\)-phase delay when the secondary wavelets, Hecht, 3.5.2, 3nd Ed)
Huygens-Fresnel principle

“Every unobstructed point of a wavefront, at a given instant in time, serves as a source of secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitude and relative phase).”

Huygens’s principle:
By itself, it is unable to account for the details of the diffraction process. It is indeed independent of any wavelength consideration.

Fresnel’s addition of the concept of interference
The Huygens–Fresnel principle. Each point on a wavefront generates a spherical wave.

The total contribution to the disturbance at \( P \) is expressed as an area integral over the primary wavefront,

\[
\psi(P) = A \frac{\exp \left[ -i(\omega t - kr_0) \right]}{r_0} \int_s \int_s \exp \left( ik_s \right) K(\chi) \, dS
\]  

(8)

Huygens’ Secondary wavelets on the wavefront surface \( S \)
**HUYGENS-FRESNEL CONSTRUCTION**: Fresnel Zones

The total contribution to the disturbance at \( P \) is expressed as an area integral over the primary wavefront,

\[
\psi(P) = A \exp \left[ -i(\omega t - kr_0) \right] \int_S \exp \left( iks \right) K(\chi) \, dS
\]

**(8)**

Spherical wave from source \( P_0 \)

\[ \text{Obliquity factor:} \]

- unity where \( \chi = 0 \) at \( C \)
- zero where \( \chi = \pi/2 \) at high enough zone index

Huygens' Secondary wavelets on the wavefront surface \( S \)

**FIGURE 2** Fresnel zone construction. \( P_0 \): point source. \( S \): wavefront. \( r_0 \): radius of the wavefront. \( b \): distance \( CP \). \( s \): distance \( QP \). (*After Born and Wolf.*)
The average distance of successive zones from \( P \) differs by \( \lambda/2 \) -> half-period zones. Thus, the contributions of the zones to the disturbance at \( P \) alternate in sign,

\[
\psi(P) = \psi_1 - \psi_2 + \psi_3 - \psi_4 + \psi_5 - \psi_6 + \ldots
\]

where \( \psi_j \) stands for the contribution of the \( j \)th zone, \( j = 1, 2, 3, \ldots \) The contribution of each annular zone is directly proportional to the zone area and is inversely proportional to the average distance of the zone to the point of observation \( P \). The ratio of the zone area to its average distance from \( P \) is independent of the zone index \( j \). Thus, in summing the contributions of the zones we are left with only the variation of the obliquity factor, \( \tilde{K}(\chi) \). To a good approximation, the obliquity factors for any two adjacent zones are nearly equal and for a large enough zone index \( j \) the obliquity factor becomes negligible. The total disturbance at the point of observation \( P \) may be approximated by

\[
\psi(P) = 1/2(\psi_1 \pm \psi_n) \quad (1/2 \text{ means averaging of the possible values, more details are in 10-3, Optics, Hecht, 2nd Ed})
\]

For an unobstructed wave, the last term \( \psi_n = 0 \).

\[
\psi(P) = 1/2\psi = \frac{A}{r_0 + b} \lambda \exp\{-i[\omega t - k(r_0 + b) - \pi/2]\}
\]

Whereas, a freely propagating spherical wave from the source \( P_0 \) to \( P \) is

\[
\psi(P) = \frac{A}{r_0 + b} \exp\{-i[\omega t - k(r_0 + b)]\} = \frac{1}{i\lambda} \left(\frac{\exp(iks)}{s}\right)
\]

Therefore, one can assume that the complex amplitude of \( \exp(iks)/s \)
**HUYGENS-FRESNEL CONSTRUCTION**

: Diffraction of light from circular apertures and disks

(a) The first two zones are uncovered,

\[ \psi(P) = \psi_1 - \psi_2 = 0! \]

(consider the point P at the on-axis P)

since these two contributions are nearly equal.

(b) The first zone is uncovered if point P is placed farther away,

\[ \psi(P) = \psi_1 \]

which is twice what it was for the unobstructed wave!

: Babinet principle

\[ \psi_s(P) + \psi_{cs}(P) = \psi_{un}(P) \]

(c) Only the first zone is covered by an opaque disk,

\[ \psi(P) = -\psi_2 + \psi_3 - \psi_4 + \psi_5 - \psi_6 + \cdots = -\frac{1}{2}\psi_2 \]

which is the same as the amplitude of the unobstructed wave.

**FIGURE 4** The redrawn zone structure for use with an off-axis point P. (After Andrews.)

**FIGURE 6** Variation of on-axis irradiance when the

**Diffraction patterns from circular apertures**
**HUYGENS-FRESNEL CONSTRUCTION** : Straight edge

\[ \psi_a(P) = 1/2 \psi_1 \]

At the edge

\[ \psi_B(P) = 1/2 \psi_1 + \psi_1 = 3/2 \psi_1 \]

Damped oscillating

\[ \psi(P) = 3/2 \psi_1 - \psi_2 \]

Monotonically decreasing

\[ \psi_c(P) = -1/2 \psi_2 \]

\[ \psi(P) = 1/2 \psi_3 \]
After the Huygens-Fresnel principle

Fresnel’s shortcomings:
He did not mention the existence of backward secondary wavelets, however, there also would be a reverse wave traveling back toward the source. He introduce a quantity of the obliquity factor, but he did little more than conjecture about this kind.

\[
\psi(P) = A \frac{\exp \left(-i(\omega t - kr_0)\right)}{r_0} \int_S \frac{\exp (iks)}{s} K(\chi) \, dS
\]

Gustav Kirchhoff: Fresnel-Kirchhoff diffraction theory
A more rigorous theory based directly on the solution of the differential wave equation. He, although a contemporary of Maxwell, employed the older elastic-solid theory of light. He found \( K(\chi) = (1 + \cos \theta)/2 \). \( K(0) = 1 \) in the forward direction, \( K(\pi) = 0 \) with the back wave.

\[
\psi(P) = -\left(\frac{ia}{2\lambda}\right) \int_A \left[ \frac{\exp (ikr)}{r} \right] \left[ \frac{\exp (iks)}{s} \right] \cos (n, r) - \cos (n, s) \, dS
\]

Arnold Johannes Wilhelm Sommerfeld: Rayleigh-Sommerfeld diffraction theory
A very rigorous solution of partial differential wave equation. The first solution utilizing the electromagnetic theory of light.

\[
\psi(P) = -\left(\frac{ia}{\lambda}\right) \int_A \left[ \frac{\exp (ikr)}{r} \right] \left[ \frac{\exp (iks)}{s} \right] \cos (n, s) \, dS
\]
Kirchhoff's theorem

Starting point: Field known on a closed surface $S$.
What is the field in a point $P_0$ inside $S$?

Scalar approximation (polarization effects ignored)

\[
\begin{align*}
\text{Gauss: } & \int_A \mathbf{A} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{A} \, dv \\
& \text{If } \mathbf{A} = G \nabla U - U \nabla G \quad (U(t) \text{ and } G(t) \text{ arbitrary scalar functions}) \\
\text{Green II: } & \int_S (G \nabla U - U \nabla G) \, dS = \int_V (G \nabla^2 U - U \nabla^2 G) \, dv
\end{align*}
\]

Assume now that $U$ and $G$ satisfy the homogeneous wave equation and that their time dependence is $\sim e^{i\omega t}$

\[
\begin{align*}
\nabla^2 U &= \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} = -k^2 U \\
\nabla^2 G &= \frac{1}{c^2} \frac{\partial^2 G}{\partial t^2} = -k^2 G
\end{align*}
\]

\[
k^2 = \frac{\omega^2}{c^2} \quad \Rightarrow \int_V (G \nabla^2 U - U \nabla^2 G) \, dv = 0
\]

Fresnel-Kirchhoff diffraction formula:

\[
\psi(P) = - \left( \frac{ia}{2\lambda} \right) \int_A \left[ \int_A \frac{\exp(ikr)}{r} \left[ \frac{\exp(iks)}{s} \right] \right] \left[ \cos(n, r) - \cos(n, s) \right] \, dS
\]

Rayleigh-Sommerfeld diffraction formula

\[
\psi(P) = - \left( \frac{ia}{\lambda} \right) \int_A \left[ \int_A \frac{\exp(ikr)}{r} \left[ \frac{\exp(iks)}{s} \right] \right] \cos(n, s) \, dS
\]
Fresnel diffraction

Assume: $z \gg x_1, y_1, x_0, y_0$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rightarrow \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \quad (U = 0 \text{ outside the aperture})$$

Fresnel's approximation:

In the exponent:

$$r = \sqrt{z^2 + (x_0 - x_1)^2 + (y_0 - y_1)^2}$$

$$\approx \left[ 1 + \frac{1}{2} \left( \frac{x_0 - x_1}{z} \right)^2 + \frac{1}{2} \left( \frac{y_0 - y_1}{z} \right)^2 \right] \quad \left( \sqrt{1 + x^2} \approx 1 + \frac{x^2}{2} \right)$$

In the denominator: $r \rightarrow \zeta$

$$U(x_0, y_0) = -\frac{ik}{2\pi} \frac{e^{ikz}}{z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x_1, y_1) e^{ik\left[ (x_0 - x_1)^2 + (y_0 - y_1)^2 \right]} \; dx_1 dy_1$$
Fraunhofer diffraction

Fraunhofer’s approximation:

\[
\frac{ik}{2z} \left[ (x_0 - x_1)^2 + (y_0 - y_1)^2 \right] = \frac{ik}{2z} \left[ (x_0^2 + y_0^2) + (x_1^2 + y_1^2) - 2(x_0x_1 + y_0y_1) \right]
\]

if

\[
\frac{k(x_1^2 + y_1^2)}{2} \ll z
\]

\[
U(x_0, y_0) = A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x_1, y_1) \ e^{-i \frac{k}{z} (x_0x_1 + y_0y_1)} \ dx_1 \ dy_1
\]

\[
A = \frac{ik}{2\pi} \frac{e^{i \frac{k}{z} (x_0^2 + y_0^2)}}{e^{2i\frac{k}{z}x_0}}
\]

\[
z \sim D^2/\lambda
\]

\[
D \sim 1 \ cm
\]

\[
\lambda \sim 1 \ \mu m
\]

\[
z \sim 100 \ m
\]

\[
U(x_0, y_0) \text{ is given by the Fourier transform of } U(x_1, y_1)
\]

Fourier optics
Examples of Fourier transforms

<table>
<thead>
<tr>
<th>Function</th>
<th>F-transform</th>
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<tbody>
<tr>
<td>Delta function</td>
<td>Const.</td>
</tr>
<tr>
<td>1-D rectangular</td>
<td>Sin x/x</td>
</tr>
<tr>
<td>function</td>
<td></td>
</tr>
<tr>
<td>2-D rectangular</td>
<td>Sin x/x</td>
</tr>
<tr>
<td>function</td>
<td></td>
</tr>
<tr>
<td>Isosceles triangle</td>
<td>(Sin x/x)^2</td>
</tr>
<tr>
<td>Gaussian</td>
<td></td>
</tr>
<tr>
<td>Gaussian</td>
<td></td>
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