The first-order photonic stopband has widely been utilized for photonic crystal applications.

At higher-order stopbands including second-order one, the coupling between GMR (leaky) band-edge states and a continuum of radiation states from the higher-order stopband can generate an confined mode, which can guide through the topological interface (edge).

There may exist a strong relation between EP in NH photonics and DP in topological photonics.

“It is worth mentioning the potential of utilizing exceptional point singularities in optical scattering problems, where the coupling between discrete localized metastable states and a continuum of radiation states is concerned. Such optical scattering problems in general, be treated as non-Hermitian problems, for which a point of interest would be to explore the connection between radiation leakage and exceptional points emerging in the continuum.”
(Mohammad-Ali Miri and Andrea Alù, Science 363, 42 (2019)
Bragg reflector and Resonance grating

Single-layer SOI resonance device

21-layer SiO₂/TiO₂ Bragg stack

\[ \eta_R \] vs. \( \lambda (\mu m) \)

Homogeneous layer with the same effective index

Periodic waveguide

Reflectance vs. Wavelength (nm)
Bragg diffraction

Bragg scattering

Regardless of how small the reflectivity $r$ is from an individual scatter, the total reflection $R$ from a semi infinite structure:

$$R = re^{-i2d} + re^{-i2d}e^{-i2d} + re^{i2d}e^{i2d} + ... = re^{-i2d} \frac{1}{1 - e^{-i2d}}$$

Diverges if

$$e^{2id} = 1 \quad k = \frac{\pi}{\alpha}$$

Bragg condition

Light cannot propagate in a crystal, when the frequency of the incident light is such that the Bragg condition is satisfied

Origin of the photonic band gap

Bragg Reflection

$$\lambda_B = 2nd \cdot \sin(\theta_B)$$

$$\lambda_B \sim 2d$$

$$k_B = \frac{2\pi}{\lambda_B} = \frac{\pi}{d}$$

Incident Beam
Scattered Beam
path-length difference $2 \cdot d \cdot n \cdot \sin \theta$

Bragg Diffraction

1. INCIDENT WAVE
2. REFLECTED WAVES IN PHASE
3. TOTAL WAVE

1. INCIDENT WAVE
2. REFLECTED WAVES NOT IN PHASE
3. TOTAL WAVE
Dispersion relation (photonic band structure)

Visualization of the vacuum band structure (2d)

For a two-dimensional system:

\[ \omega = c \sqrt{k_x^2 + k_y^2} \]

This function depicts a cone: light cone.

A few ways to visualize this band structure:

- Constant frequency contour
- Projected band diagram
- Band diagram along several “special” directions

Photonic Bandgap Formation

1. Dispersion curve for free space

2. In a periodic system, when half the wavelength corresponds to the periodicity

\[ \frac{\lambda}{2} = a \quad k = \frac{\pi}{a} \]

the Bragg effect prohibits photon propagation.

3. At the band edges, standing waves form, with the energy being either in the high or the low index regions

4. Standing waves transport no energy with zero group velocity

Band-edges

Air band
standwave in \( n_2 \)

Bandgap

Dielectric band
standwave in \( n_1 \)

Reduced (1st Brillouin zone) band structure

\[ G = \frac{2\pi}{d} \]

\[ G = \frac{2\pi}{d} \]
Guided-mode resonance (GMR) from symmetric and asymmetric grating profiles

(a) Grating with symmetric profile

(b) Grating with asymmetric profile

Details: Y. Ding and R. Magnusson, “Use of nondegenerate resonant leaky modes to fashion diverse optical spectra,” Optics Express, May 3, 2004
For symmetrical grating (Type I):

For asymmetrical grating (Type II):

Use of nondegenerate resonant leaky modes to fashion diverse optical spectra

Y. Ding and R. Magnusson

Opt Express 12, 1885 (2004)
IEEE J QE
QE-21, 144 (1985)

Second-Order Distributed Feedback Lasers with Mode Selection Provided by First-Order Radiation Losses
RUDOLF F. KAZARINOV AND CHARLES H. HENRY

Stop bands in Symmetric profiles

→ The dispersion curve resulting from the coupling coefficient $h_2$, for the case in which losses are neglected ($h_1 = \alpha - g = 0$).

(Second-order) grating vector ($K_0$) = propagation vector

→ The fundamental transverse mode for the waveguide without the periodic grating, with real dielectric function $\varepsilon(x, z)$, propagation vector $K_0$.

The dielectric function of the waveguide

$\varepsilon(x, z) = \varepsilon_0(x) + \Delta \varepsilon(x, z)$

The grating layer

$\Delta \varepsilon(x, z) = -\Delta \varepsilon \sum_{m \neq 0} \xi_m(x) \exp(-iK_0 mz)$.

For a grating with reflection symmetry:

$\Delta \varepsilon(x, z) = \Delta \varepsilon(x, -z)$.

$\xi_m = \xi - m!$

At one edge of the gap the phase difference is 180° → the radiation cancels,

At the other edge the phase difference is 0° → the radiation loss is enhanced.
The wave equation is given by

\[ E = \left[ A(z) \exp(\text{i}K_0 z) + B(z) \exp(-\text{i}K_0 z) \right] \cdot \phi(x) + \Delta E(x, z). \]

Setting the sum of terms with the same \( \exp(\text{i}mK_0 z) \) \((m = 1, 0, -1)\) to zero,

\[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} \varepsilon_0(x) - \frac{\omega^2}{c^2} \Delta \varepsilon \sum_m \xi_m(x) \cdot \exp(\text{i}K_0 mz) E(x, z) = 0. \]

The sum of the responses from the source points \( y \)

\[ \phi(x) \rightarrow \text{the fundamental transverse mode for the waveguide} \]

\[ \phi(x) \text{ satisfies } \left[ \frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2} \varepsilon_0(x) - K_0^2 \right] \phi(x) = 0. \]

\[ (2) \& (3) \text{ the dispersion relation,}\]

relating changes in wavevector, changes in angular frequency

\[ \left( \frac{\Delta \omega}{v_g} + i \frac{\alpha - g}{2} - i \frac{d}{dz} \right) A + h_2 B + ih_1 (A + B) = 0 \]

\[ \left( \frac{\Delta \omega}{v_g} + i \frac{\alpha - g}{2} + i \frac{d}{dz} \right) B + h_2 A + ih_1 (A + B) = 0 \]

by finding Green's function \( G(x, y) \) satisfying \( G \sim \text{Impulse response function} \)

\[ (1) \rightarrow \left[ \frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2} \varepsilon_0(x) - K_0^2 \right] G(x, y) = \delta(x - y). \]

\[ \Delta E(x, z) = \frac{\omega^2 \Delta \varepsilon}{c^2} \int G(x, y) \xi_1(y)(A + B) \cdot \phi_0(y) \, dy. \]

\[ \rightarrow \text{The sum of the responses from the source points} \]
Stop bands in Symmetric profiles \( \xi_m = \xi_{-m} \):

\[
(4) \quad \left( \frac{\Delta \omega}{v_g} + i \left( \frac{\alpha - g}{2} + i \frac{d}{dz} \right) \right) A + h_2 B + i h_1 (A + B) = 0 \quad \rightarrow \quad \left[ \frac{\Delta \omega}{v_g} + i \left( \frac{\alpha - g}{2} + h_1 \right) + i \frac{d}{dz} \right] A + (h_2 + i h_1) B = 0 \\
(5) \quad \left( \frac{\Delta \omega}{v_g} + i \left( \frac{\alpha - g}{2} - i \frac{d}{dz} \right) \right) B + h_2 A + i h_1 (A + B) = 0 \quad \rightarrow \quad (h_2 + i h_1) A + \left[ \frac{\Delta \omega}{v_g} + i \left( \frac{\alpha - g}{2} + h_1 \right) - i \frac{d}{dz} \right] B = 0
\]

By substitution \( A(z) \sim B(z) \sim \exp(ikz) \). \( \rightarrow k = 0 \) we have standing waves with zero group velocity.

The dispersion relation is

\[
(6) \quad \frac{\Delta \omega}{v_g} = i \left( \frac{g - \alpha}{2} \right) - ih_1 \pm [(h_2 + ih_1)^2 + k^2]^{1/2} \rightarrow \text{If we neglect absorption and radiation losses,} \ (h_1 = \alpha - g = 0).
\]

\[ \rightarrow \text{The dispersion spectrum is} \quad \frac{\Delta \omega}{v_g} = \pm (h_2^2 + k^2)^{1/2} \]

\[ \rightarrow \text{When losses are present, at} \ k = 0 \ (\text{Band edges}), \quad \text{The dispersion spectrum is} \quad \frac{\Delta \omega}{v_g} = -ih_1 \pm (h_2 + ih_1) \rightarrow \text{No radiation loss} \]

The mode field is the sum of two oppositely propagating Bloch waves

\[ \frac{\Delta \omega}{v_g} = i \left( \frac{g - \alpha}{2} \right) - ih_1 \pm [(h_2 + ih_1)^2 + k^2]^{1/2} \rightarrow \text{In general, the solution has two equal and opposite values of} \ k \]

\[
A(z) = \alpha \exp(ikz) + \beta R(k) \exp(-ikz) \\
B(z) = \alpha R(k) \exp(ikz) + \beta \exp(-ikz) \\
R = B/A = \frac{-h_2 + ih_1}{\pm [(h_2 + ih_1)^2 + k^2]^{1/2} + k} \quad |R(k)| \leq 1
\]

\[
E = [A(z) \exp(iK_0 z) + B(z) \exp(-iK_0 z)] = \alpha [1 + R(k) \exp(-2iK_0 z)] \exp\left[i(K_0 + k) z\right] + \beta [1 + R(k) \exp(2iK_0 z)] \exp[-i(K_0 + k) z] = \alpha \exp(ikz) + \beta R \exp(-ikz) \exp(iK_0 z) + \alpha R \exp(ikz) + \beta \exp(-ikz) \exp(-iK_0 z).
\]
Band gaps and leaky-wave effects in resonant photonic-crystal waveguides

Y. Ding and R. Magnusson

Opt Express
15, 680 (2007)

The dispersion relation of the second-order band:

\[
\frac{\Delta \omega}{V_g} = i \left( \frac{g - \alpha}{2} \right) - \frac{i \hbar \Delta \gamma}{2} \pm \left[ \left( \hbar_2 + i \hbar_1 \right)^2 + k^2 \right]^{1/2}
\]

net gain \( g - \alpha s = 0 \)

\( \Delta \beta - \beta K = k \)

\( \Delta k = k_0 - k_{\text{center}} = \Delta \omega \)

\( \Delta \beta = \Delta k h_3 \)

\( h_3 \) is a coefficient related to the group velocity

Or, in a different form,

\[
\left( \Delta k \cdot h_3 + j \hbar_1 \right)^2 = \Delta \beta^2 + \left( \left| h_2 \right|^2 + 2 j \hbar_1 \Re \left( h_2 \right) - h_1^2 \right)
\]

The grating layer

\[
\Delta e(x, z) = - \Delta e \sum_{m \neq 0} \gamma_m(x) \exp(-iK_0 mz).
\]

For a binary grating, \( \gamma_m(x) = \gamma_0 \) are constant.
Band gaps and leaky-wave effects in resonant photonic-crystal waveguides

Y. Ding and R. Magnusson


In the case when \( h_1 = 0 \) (the nonleaky band edge of the second-order band)

\[
\Delta \beta^2 = \left[ (\Delta k \cdot h_3) - \left| h_2 \right|^2 \right] + 2 j h_1 \left[ (\Delta k \cdot h_3) - \text{Re}(h_3) \right]
\]

\[
\Delta k = \pm \sqrt{(\Delta \beta)^2 + \left| h_2 \right|^2} / h_3
\]

\( \rightarrow k(\omega) \) real freq; \( \beta \) complex prop.

\( \rightarrow \beta \) real prop; \( k(\omega) \) complex freq.

In the case when \( h_1 \) is not zero (the leaky band edge)

At frequency \( \Delta k = h_2 / h_3 \)

\( \beta \) propagation constant has an imaginary part

At frequency \( \Delta k = -h_2 / h_3 \)

\( k(\omega) \) real freq; \( \beta \) complex prop.

At frequencies far from the band edges

\( |\Delta k| >> h_2 / h_3 \)

\( \Delta \beta = \sqrt{(\Delta k h_3)^2 - h_2^2} + j h_1 \)

\( \beta_l = \text{Re}(h_3) / K \)
Band gaps and leaky-wave effects in resonant photonic-crystal waveguides

Y. Ding and R. Magnusson

(1) Band structure in real frequency

\[ \Delta \beta = \pm \sqrt{\left( \Delta k : h_3 \right)^2 - |h_2|^2} \]

\[ \Delta k = \frac{h_2}{h_3} \]

At frequency \( \Delta k = \frac{h_2}{h_3} \) \( \Rightarrow \Delta \beta = 0 \)
\( \beta_i = 0 \)
\( \Rightarrow \) the upper band edge
\( \Rightarrow \) \( h_1 \) has no effect
\( \Rightarrow \) Nonleaky band edge

At frequency \( \Delta k = -\frac{h_2}{h_3} \)
\( \Delta \beta = \Delta \beta_R + j \Delta \beta_i \)
\( = \sqrt{-4 j h_2 h_1} \)
\( \Rightarrow \) Leaky edge

At frequencies far from the band edges
\( |\Delta k| >> \frac{h_2}{h_3} \)
\( \Delta \beta = \sqrt{\left( \Delta k h_3 \right)^2 - h_2^2 + h_1} \)
\( \beta_i = \text{Re}(h_i) / K \)

(2) Band structure in real propagation constant

\[ \Delta k = \pm \sqrt{\left( \Delta \beta \right)^2 + \frac{|h_2|^2}{h_3}} \]

At frequency \( \Delta k = \frac{h_2}{h_3} \) \( \Rightarrow k_0 \times 0 \)
\( \Rightarrow \) Nonleaky band edge

At frequency \( \Delta k = -(h_2 + 2 j h_1)/h_3 \)
\( k_0 \times -2 \text{Re}(h_1)/h_3 \)

At frequencies far from the band edges
\( \Delta k = (\pm \Delta \beta - j h_1)/h_3 \)
\( k_0 \times -\text{Re}(h_1)/h_3 \)
Band gaps and leaky-wave effects in resonant photonic-crystal waveguides

Y. Ding and R. Magnusson

15, 680 (2007)

Opt Express

(1) Band structure in real frequency

$$\Delta \beta = \pm \sqrt{(\Delta k \cdot h_3)^2 - |h_2|^2}$$

⇒ $k(\omega)$ real frequency, $\beta$ complex

(2) Band structure in real propagation constant

$$\Delta k = \pm \sqrt{(\Delta \beta)^2 + |h_2|^2}$$

⇒ $k(\omega)$ complex frequency, $\beta$ real

<table>
<thead>
<tr>
<th>Band edge</th>
<th>$\Delta \beta_R$</th>
<th>$\Delta \beta_I$</th>
<th>$\Delta k_R$</th>
<th>$\Delta k_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band edge 1 (upper edge in the example treated)</td>
<td>$0$</td>
<td>$0$</td>
<td>$\frac{h_z}{h_3}$</td>
<td>$\frac{h_z}{h_3}$</td>
</tr>
<tr>
<td></td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Band center</td>
<td>$\text{Re}\left(-\frac{h_z^2}{2} - 2j\beta h_z\right)$</td>
<td>$0$</td>
<td>$\text{Im}\left(-\frac{h_z^2}{2} - 2j\beta h_z\right)$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$0$</td>
<td>No solution</td>
<td>$0$</td>
<td>No solution</td>
</tr>
<tr>
<td>Band edge 2 (lower edge in the example treated)</td>
<td>$\text{Re}\left(-4j\beta h_z\right)$</td>
<td>$0$</td>
<td>$\text{Im}\left(-4j\beta h_z\right)$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$-\frac{h_z}{h_3}$</td>
<td>$-\frac{h_z}{h_3}$</td>
<td>$-2\text{Re}(h_z)/h_3$</td>
<td>$-2\text{Re}(h_z)/h_3$</td>
</tr>
<tr>
<td>Away from band</td>
<td>$\text{Re}\left((\Delta \beta h_z)^2 - h_z^2\right)$</td>
<td>$\approx 0$</td>
<td>$\text{Re}(h_z)$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td></td>
<td>$\text{Im}(\Delta \beta h_z)$</td>
<td>$\approx 0$</td>
<td>$\pm \sqrt{\Delta \beta^2 + h_z^2}/h_z$</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td></td>
<td>$-2\text{Re}(h_z)/h_3$</td>
<td>$-\text{Re}(h_z)/h_3$</td>
<td>$-\text{Re}(h_z)/h_3$</td>
<td>$-\text{Re}(h_z)/h_3$</td>
</tr>
</tbody>
</table>

⇒ The dispersion plots in complex frequency give a more clearly defined band gap.
Phase matching condition to excite a resonance with a plane wave incident at angle $\theta$,

$$k_0 n_c \sin \theta - m K = \text{Re}(\beta_v)$$

The representation in complex frequency delivers the real propagation constant directly and matches the results from the diffraction computations.

$$k (\omega) \text{ complex frequency, } \beta \text{ real}$$
Use of nondegenerate resonant leaky modes to fashion diverse optical spectra

Y. Ding and R. Magnusson

12, 1885 (2004)

Second stopband characteristics

Asymmetric profile

Possible interaction between GMRs #2 & 3 as Δε increases. Since each GMR is associated with 100% reflection, placing two GMRs near each other opens the possibility of a flat reflection band.
Use of nondegenerate resonant leaky modes to fashion diverse optical spectra

Y. Ding and R. Magnusson

Example 1: Bandstop filter with narrow flattop

Example 2: Bandstop filter with wide flattop

Example 3: Bandpass filter
Use of nondegenerate resonant leaky modes to fashion diverse optical spectra

Y. Ding and R. Magnusson

Example 2: Bandstop filter with wide flattop

Example 3: Bandpass filter

Possible interaction between GMRs #2 & 3 as $\Delta\varepsilon$ increases. Since each GMR is associated with 100% reflection, placing two GMRs near each other opens the possibility of a flat reflection band.
Bandpass filters for TE polarization

Two TE\(_{11}\) modes (GMR\#3, GMR\#4)

- \(n_a=1.0\)
- \(n_h=3.48\)
- \(n_{\text{low}}=1.0\)
- \(n_s=1.48\)
- \(\lambda=1.0\ \text{μm}\)
- \(d=0.7\ \text{μm}\)
- \(F_1=0.1\)
- \(F_2=0.35\)
- \(M=0.35\)

FWHM passband of 3.5 nm

The transmission peak is provided by the asymmetrical lineshape of TE\(_{11}\) at the upper stopband edge which generates the corresponding minima in the \(\eta_R\) curve near \(\lambda=1.65\ \text{μm}\).
Resonant leaky-mode spectral-band engineering and device applications

Second stopband characteristics

Bandpass filters for TM polarization

Asymmetric profile

(Mirror) Wideband reflector for TE polarization

Symmetric profile

The profile is symmetric in this example, only one GMR will exist for each stopband. Since the total effective fill factor is less than 0.5 (F=0.45), the GMR appears at the lower bandedge and concentrates in the high-ε region.

→ No GMR#2, GMR#4 (low-ε region) since symmetry
Second stopband characteristics

(Mirror) Wideband reflector for TM polarization

the profile is symmetric in this example, only one GMR will exist for each stopband. Since the total effective fill factor is less than 0.5 (F=0.45), the GMR appears at the lower bandedge and concentrates in the high-ε region.
Second stopband characteristics

Polarizers

Asymmetric profile

TE reflectance

TM transmittance

GMR#2 (TE\textsubscript{1,0})

GMR#3 (TM\textsubscript{1,1})

GMR#4 (TM\textsubscript{1,1})

GMR#3 (TM\textsubscript{1,1})

GMR#4 (TM\textsubscript{1,1})

GMR#4 (TM\textsubscript{1,1})

GMR#3 (TM\textsubscript{1,1})

GMR#2 (TM\textsubscript{1,0})

(Short wavelength)

(Short wavelength)

(Short wavelength)

(Short wavelength)
Resonant leaky-mode spectral-band engineering and device applications

Second stopband characteristics

Polarization independent element
Asymmetric profile

Antireflection element
Asymmetric profile

TE TM reflectance

TE TM Transmittance

TM reflectance

Asymmetric profile
Time-delay elements based on leaky-mode resonance for optical buffers, delay lines, and switches.

A transform-limited TE-polarized Gaussian pulse is

\[ E_y(t) = E_0 \exp\left[-\frac{(t-t_0)^2}{T^2}\right]\exp[j\omega_0(t-t_0)] \]

The time delay (\(\tau\)) and delay dispersion (D) are

\[ \tau = \left(\frac{\lambda^2}{2\pi c}\right)d\phi/d\lambda \]

\(\phi \rightarrow \lambda\) dependent phase shift after reflection or transmission

\[ D = d\tau/d\lambda \]

\(\Lambda = 979\) nm, \(d = 465\) nm,
\(n_H = 3.48, n_S = 1.48,\) and \(n_L = n_{\text{inc}} = 1.0\)

\([F_1, F_2, F_3, F_4] = [0.071, 0.275, 0.399, 0.255]\)
Dispersion Engineering with Leaky-Mode Resonant Photonic Lattices

Y. Ding and R. Magnusson

\[ \tau = \frac{\lambda^2}{2\pi c} \frac{d\phi}{d\lambda} \]

\[ D = \frac{d\tau}{d\lambda} \]

**\( \Lambda = 1103.9 \text{ nm}, d = 432.2 \text{ nm}, d_{\text{cavity}} = 2000 \text{ nm} \)**

**\( n_H = 3.48, n_S = 1.48, \text{ and } n_L = n_{\text{inc}} = 1.0 \)**

\([F_1, F_2, F_3, F_4] = [0.0626, 0.3013, 0.4576, 0.1785] \]

an average delay of \(~7 \text{ ps}\)
Leaky-mode resonant reflectors with extreme bandwidths
multilevel resonant leaky-mode structures
Mehrdad Shokooh-Saremi and R. Magnusson


\[ \Lambda = 846.4 \text{ nm}, \]
\[ n_H = 3.48, \quad n_S = 1.48, \quad \text{and} \quad n_L = n_{\text{inc}} = 1.0 \]
\[ d_1 = 375 \text{ nm}, \quad d_2 = 175 \text{ nm}, \quad d_3 = 375 \text{ nm} \]
\[ F_{11} = F_{31} = 0.283, \quad F_{21} = 1.0 \]

\[ \Lambda = 882.4 \text{ nm}, \]
\[ n_H = 3.48, \quad n_S = 1.48, \quad \text{and} \quad n_L = n_{\text{inc}} = 1.0 \]
\[ d_1 = 572 \text{ nm}, \quad d_2 = 455 \text{ nm}, \quad d_3 = 572 \text{ nm} \]
\[ F_{11} = F_{21} = F_{31} = 0.417 \]

\[ n_{\text{GE}} = 4.0 \]
Guided-mode resonant wave plates

Mehrdad Shokooh-Saremi and R. Magnusson


Half-wave plate (Si)

\[ \Lambda = 786.8 \text{ nm}, \]
\[ n_H = 3.48, n_S = 1.48, \text{ and } n_i = n_{inc} = 1.0 \]
\[ d_1 = 525.3 \text{ nm}, \quad d_2 = 624.6 \text{ nm} \]
\[ F = 0.2665 \]

Fabrication tolerance

- deviation (5.0%) in silicon thickness
- deviation in grating fill factor (F)

\[ \Rightarrow \text{The retarder is rather sensitive to deviations in Si thickness.} \]
Half-wave plate \((\text{Si} + \text{HfO}_2)\)

\[ \Lambda = 780.9 \text{ nm}, \]
\[ n_{\text{Si}} = 3.48, \quad n_{\text{HfO}_2} = 2.0, \quad n_S = 1.48, \quad \text{and} \quad n_L = n_{\text{inc}} = 1.0 \]
\[ d_{\text{HfO}_2} = 1086.2 \text{ nm}, \quad d_{\text{Si}} = 375.3 \text{ nm} \]
\[ F = 0.4876 \]

Quarter-wave plate \((\text{Si} + \text{HfO}_2)\)

\[ \Lambda = 878.9 \text{ nm}, \]
\[ n_{\text{Si}} = 3.48, \quad n_{\text{HfO}_2} = 2.0, \quad n_S = 1.48, \quad \text{and} \quad n_L = n_{\text{inc}} = 1.0 \]
\[ d_{\text{HfO}_2} = 586.1 \text{ nm}, \quad d_{\text{Si}} = 413.8 \text{ nm} \]
\[ F = 0.3072 \]
A generalized theoretical analysis on the tunable characteristics of the GMRs in coupled gratings

Let's investigate the general resonance characteristics of the coupled resonator by using the coupled-mode theory.

\[
\frac{da_1}{dt} = \left(j\omega_0 - \frac{2}{\tau}\right)a_1 - j\mu a_2 + \kappa s_{a_1} + \kappa s_{a_2},
\]

\[
\frac{da_2}{dt} = \left(j\omega_0 - \frac{2}{\tau}\right)a_2 - j\mu a_1 + \kappa s_{a_3} + \kappa s_{a_4},
\]

(\(\kappa\)) Radiation coupled via the leakage wave (outgoing wave)

\[
\kappa = e^{j\phi} \sqrt{\frac{2}{\tau}} \quad \rightarrow \quad \text{Radiation coupling coefficient}
\]

\[
S_{+i} \rightarrow \text{the incoming wave amplitudes}
\]

\[
S_{-i} \rightarrow \text{the outgoing wave amplitudes}
\]

\[
1/\tau \quad \text{is a decay rate:}
\]

\[
Q = \omega_0/4
\]

(\(\mu\)) Direct (evanescent) coupled

\[
\mu \rightarrow \text{Direct (evanescent) coupling coefficient}
\]

\[
\gamma_e \rightarrow \kappa \quad \text{(radiation coupling coefficient)}
\]

\[
\gamma_i \rightarrow \mu \quad \text{(evanescent coupling coefficient)}
\]
Because of energy conservation and time reversal symmetry, $\mu$ is real.

\[
\begin{align*}
S_2 &= s_1 - \kappa^2 a_1 \\
S_1 &= s_2 - \kappa^2 a_1 \\
S_3 &= s_4 - \kappa^2 a_2 \\
S_4 &= s_3 - \kappa^2 a_2
\end{align*}
\]

Define the amplitudes of the super-modes with even and odd symmetries

\[
\begin{align*}
a_{\text{even}} &= \frac{a_1 + a_2}{\sqrt{2}} \quad \text{and} \quad a_{\text{odd}} = \frac{a_1 - a_2}{\sqrt{2}}
\end{align*}
\]

Reflection coefficient (when $S_{+4} = 0$)

\[
r = \frac{s_{-1}}{s_{+1}} = \left| \frac{-j \left( \frac{(\omega - \omega_o)\tau}{2} + j\mu \tau e^{-j\theta} - 1 + e^{-j2\theta} \right)}{j \left( \frac{(\omega - \omega_o)\tau}{2} + 1 \right)^2 - j \left( \frac{\mu \tau}{2} + e^{-j\theta} \right)^2} \right|
\]
Reflection coefficient (when $S_{++} = 0$)

\[
r = \frac{S_{+}}{S_{-}} = \frac{-j(\theta + \omega_0)\tau}{2(1 + e^{-j\theta})} + j\mu e^{i\theta} \left(\frac{1}{2} - 1 + e^{-j\theta}\right)
\]

For \(0 < \theta < \pi\), the odd super-mode gets broader and the even super-mode peak gets sharper.

For a strong direct evanescent coupling regime, \((\mu \gg 1/\tau, \mu \gg |k|)\), the resonance frequency tuning is dominantly governed by \(\mu\). The phase retardation \(\theta\) does not change the resonance peaks much but mainly affects the linewidths of the peaks.

\(s (0 \rightarrow \pi/2)\) means the phase retardation \(\theta \rightarrow \pi\). the Zak phase?

For \(0 < \theta < \pi\), the odd super-mode gets broader and the even super-mode peak gets sharper.

For \((\mu \rightarrow 0)\) such that evanescent coupling is negligible, the phase retardation \(\theta\) plays an important role.
In strong evanescent coupling regime

For a single grating normally incidence (TE), a narrow linewidth of 1.5 nm ($\lambda_0 = 1599.2$ nm)

d ($0 \rightarrow 500$ nm) means the evanescent coupling strength ($\mu$) $\rightarrow 0$.

s ($0 \rightarrow P/2$) means the phase retardation ($\theta$) $0 \rightarrow \pi$. the Zak phase??

Odd

Even

Odd

Even

Odd
In strong evanescent coupling regime

\[ \theta \to 0 \] the Odd modes get sharp.

\[ \theta \to \pi \] the Even modes get sharp.

\[ s \to P/4 \] means the phase retardation \((\theta \to \pi/2)\).

A very sharp reflection dip (a transmission peak) is observed near the even mode peak for \(d < 100\) nm. The dip results from a destructive interference between two nondegenerate GMRs associated with the even mode. In an asymmetric waveguide grating, both edges of the second stop band support GMRs. Interface mode?? CHECK MODE PROFILE!!

\[ s \to P/2 \] means the phase retardation \((\theta \to \pi)\). The even width \(\to 0\). The effect of the gratings on the field is very small.
In negligible evanescent coupling regime

The calculated spectra are independent of the horizontal alignment of two grating since free space propagation is a dominant coupling mechanism.
Experimental and Theoretical Demonstration of Resonant Leaky-Mode in Grating Waveguide Structure with A Flattened Passband

National Central University, Taiwan

Poly-Si (3.48)
SiO2 (1.46)

\[ \Lambda = 700 \text{ nm} \]
\[ d_p = 400 \text{ nm} \]
\[ d_w = 100 \text{ nm} \]

Lateral shift: 0.5 \( \Lambda \)

f. factors: 0.12 and 0.38
Evolution of modes of Fabry–Perot cavity based on photonic crystal guided-mode resonance mirrors

Pierre Pottier, Lina Shi, and Yves-Alain Peter, Ecole Polytechnique de Montréal

Filled dots: high transmission peaks (>90%), empty dots: lower transmission peaks (<90%).

\[ g = \frac{\lambda}{2n_1} \]

\[ t_1 = \frac{(2m_1 + 1)i}{4n_1} \]

\[ t_2 = \frac{(2m_2 + 1)i}{4n_2} \]
Tunable transmission filters based on double subwavelength periodic membrane structures with an air gap

Tian Sang1,2, Tuo Cai1, Shaohong Cai2 and Zhanshan Wang, China

GaAs (3.5) \( \Lambda = 631.4 \text{ nm} \)
\( d_1 = 280 \text{ nm} \)
\( d_2 = 60 \text{ nm} \)
\( f.f = 0.18 \)

\( S = 0.0 \), \( d_g = 80 \text{ nm} \)
\( S = 0.31 \), \( d_g = 80 \text{ nm} \)
\( S = 0.31 \), \( d_g = 450 \text{ nm} \)

Single GMR