10. Fiber Optics

Last Lecture
- Fourier analysis
- Temporal coherence and line width
- Partial coherence
- Spatial coherence

This lecture
- Numerical aperture of optical fiber
- Allowed modes in fibers
- Attenuation
- Distortion
  - Modal distortion
  - Material dispersion
  - Waveguide dispersion
Optical communication systems

Sound: microphone
Visual: video camera
Data: computer

Message input → Modulator → Carri er source → LED/LD → Electrical → light

Optic fiber → Plastic/glass

Light → electrical

Detector → Signal processor → Message output

PIN APD

Amplification filtering demodulation

Attenuation and distortion

Optical-fiber cable
AM, FM, and Digital

Signal

Carrier

AM modulated

Signal

Carrier

FM modulated

Digital pulse modulated

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Figure 1.2-16  Guiding light: (a) lenses; (b) mirrors; (c) total internal reflection.
Optical fibers

Figure 2.1: Cross section and refractive-index profile for step-index and graded-index fibers.
10-4. Optics of propagation

Step index fiber

\[ n_0 \sin \theta_m = n_1 \sin \theta_m' \]
\[ \sin \varphi_c = \frac{n_2}{n_1} \]
\[ \theta_m' = 90^\circ - \varphi_c \]
\[ \sin^2 \varphi_c + \cos^2 \varphi_c = 1 \]

Numerical aperture (N.A.)

\[ N. A. = n_0 \sin \theta_m \]
\[ = n_1 \cos \varphi_c = \sqrt{n_1^2 - n_2^2} \]
Skip distance:  \[ L_s = d \cot \theta' = d \sqrt{\left(\frac{n_1}{n_0 \sin \theta} \right)^2 - 1} \]

<table>
<thead>
<tr>
<th>Core/cladding</th>
<th>(n_0)</th>
<th>(n_1)</th>
<th>(n_2)</th>
<th>(\varphi_c)</th>
<th>(\theta_{max})</th>
<th>N.A.</th>
<th>(1/L_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass/air</td>
<td>1</td>
<td>1.50</td>
<td>1.0</td>
<td>41.8°</td>
<td>90.0°</td>
<td>1</td>
<td>8944</td>
</tr>
<tr>
<td>Plastic/plastic</td>
<td>1</td>
<td>1.49</td>
<td>1.39</td>
<td>68.9°</td>
<td>32.5°</td>
<td>0.54</td>
<td>3866</td>
</tr>
<tr>
<td>Glass/plastic</td>
<td>1</td>
<td>1.46</td>
<td>1.40</td>
<td>73.5°</td>
<td>24.5°</td>
<td>0.41</td>
<td>2962</td>
</tr>
<tr>
<td>Glass/glass</td>
<td>1</td>
<td>1.48</td>
<td>1.46</td>
<td>80.6°</td>
<td>14.0°</td>
<td>0.24</td>
<td>1657</td>
</tr>
</tbody>
</table>

Note: The reciprocal of the skip distance (\(1/L_s\), or skips per meter) is calculated for a fiber of diameter 100 \(\mu\)m and at \(\theta = \theta_{max}\).
10-5. Allowed modes: in slab (planar) waveguides

Modes (self-sustaining waves) must satisfy the condition.

\[ \frac{\Delta}{\lambda} + 2\phi_r = 2m\pi \]

\[ \Delta = AB + BC = A'B + BC = 2n_1d \cos \varphi \]

\[ m = \frac{2n_1d \cos \varphi}{\lambda} + \frac{\phi_r}{\pi} \approx \frac{2n_1d \cos \varphi_m}{\lambda} \], since \( \phi_r < \pi \) (See p. 501)

- \( m = 0 \); “straight-through mode” at \( \varphi = 90^\circ \).
- Large \( m \); “high-order mode” with \( \varphi \) near \( \varphi_c \)

Total number of propagating modes is the value of \( m \) when \( \varphi_m = \varphi_c \)

\[ m_{\text{max}} = \frac{2d}{\lambda} n_1 \cos \varphi_c + 1 = \frac{2d}{\lambda} (N.A.) + 1 = \frac{2d}{\lambda} \sqrt{n_1^2 - n_2^2} + 1 \]
Allowed modes : in fibers

\[ m_{\text{max}} = \frac{1}{2} \left( \frac{\pi d}{\lambda} \right)^2 N.A. \]

\[ m_{\text{max}} = \frac{2d}{\lambda} (N.A.) + 1 \quad \text{for slabs} \]

Single-mode (mono-mode) fiber when:

\[ m_{\text{max}} < 2 \quad \rightarrow \quad \frac{d}{\lambda} < \frac{2}{\pi (N.A.)} \]

A more careful analysis:

\[ \frac{d}{\lambda} < \frac{2.4}{\pi (N.A.)} \]

Example

Suppose an optical fiber (core index of 1.465, cladding index of 1.460) is being used at \( \lambda = 1.25 \) µm. Determine the diameter for monomode performance and the number of propagating modes when \( d = 50 \) µm.

Solution

The N. A. is then \( \sqrt{(1.465^2 - 1.46^2)} = 0.121 \)

For single-mode, \( d < \frac{2.4}{\pi (0.121)} (1.25 \text{ µm}) \) or \( d < 7.9 \) µm

if \( d = 50 \) µm,

\[ m_{\text{max}} = \frac{1}{2} \left[ \pi \frac{50}{1.25 (0.121)} \right]^2 = 115 \]
10-6. Attenuation (loss)

Extrinsic losses: bending, defects, ...
Intrinsic losses: absorption, Rayleigh scattering, ...
Attenuation (absorption) coefficient

\[ \alpha_{db} = 10 \log_{10} \left( \frac{P_1}{P_2} \right) \]

[db/km]
Modal distortion

Consider two extreme modes when \( m = 0 \) and \( m = m_{\text{max}} \)

\[
\sin \varphi_c = \frac{n_2}{n_1} = \frac{\ell}{\ell'} = \frac{L}{L'}
\]

\[
\delta \tau = \tau_{\text{max}} - \tau_{\text{min}} = \frac{L'}{v} - \frac{L}{v} = \frac{L}{v} \left( \frac{n_1}{n_2} - 1 \right)
\]

Modal distortion: (step-index fiber)

\[
\delta \left( \tau \right) = \frac{n_1}{c} \left( \frac{n_1 - n_2}{n_2} \right)
\]

[sec/km]
GRIN (graded index) fibers

\[ n(r) = n_1 \sqrt{1 - 2 \left( \frac{r}{a} \right)^\alpha \Delta}, \quad 0 \leq r \leq a \]

where \( \Delta \equiv (n_1 - n_2)/n_1 \) and \( n_1 = [n(r)]_{\text{max}} \)

Modal distortion:

 modal distortion: \( \delta \left( \frac{\tau}{L} \right) = \frac{n_1}{2c} \Delta^2 = \frac{\Delta}{2} \delta \left( \frac{\tau}{L} \right)_{sr} \)

\[ \Delta/2 = 1/292 \]

, where \( n_1 = 1.46 \) and \( n_2 = 1.45 \)
Material dispersion

Pulse broadening

Dispersion in fused quartz
Refractive index versus wavelength

- Refractive index
- Wavelength (μm)

Graph showing the relationship between refractive index and wavelength for fused quartz.
Material dispersion

\[ \tau(\omega) = \frac{L}{\nu_g(\omega)} \text{ where } \nu_g(\omega) = \frac{dw}{dk} \]

If the signal bandwidth is \( \Delta\omega \), the spread in arrival times per unit distance is

\[ \delta\left(\frac{\tau}{L}\right) = \frac{d}{d\omega}\left(\frac{1}{\nu_g}\right) \Delta\omega = \frac{d^2 k}{d\omega^2} \Delta\omega \]

\[ k = \frac{2\pi}{\lambda} = n\omega/c \rightarrow \frac{dk}{d\omega} = \frac{1}{c} \left( n + \omega \frac{dn}{d\omega} \right) = \frac{1}{c} \left( n - \lambda \frac{dn}{d\lambda} \right) \]

\[ \delta\left(\frac{\tau}{L}\right) = \frac{d}{d\omega}\left(\frac{dk}{d\omega}\right) \Delta\omega = \frac{d}{d\lambda}\left(\frac{dk}{d\omega}\right) \Delta\lambda = \frac{1}{c} \left( \frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda \frac{d^2 n}{d\lambda^2} \right) \Delta\lambda \]

\[ \delta\left(\frac{\tau}{L}\right) = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} \Delta\lambda \equiv -M \Delta\lambda \]

\( M : \text{ps/nm-km} \)

: temporal pulse spread (psec) per unit spectral width (nm) per unit length (km)
Waveguide dispersion

For a certain guided mode, the dependence of $\varphi_m = \varphi_m(\lambda)$ induces a pulse broadening.

For a guided mode, the effective refractive-index of the mode is

$$n_{\text{eff}} \equiv \frac{c}{v_g} = n_1 \sin \varphi_m \quad \rightarrow \quad n_1 \geq n_{\text{eff}} \geq n_1 \sin \varphi_c = n_2$$

\[ \delta \left( \frac{\tau}{L} \right) = -\frac{\lambda}{c} \frac{d^2 n_{\text{eff}}}{d\lambda^2} \Delta \lambda \equiv -M' \Delta \lambda \]

Waveguide dispersion

\[ < \delta \left( \frac{\tau}{L} \right) = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} \Delta \lambda \equiv -M \Delta \lambda \]

material dispersion

Waveguide dispersion is a small effect that becomes important only when the other pulse-broadening effects have been essentially eliminated. However, its presence is important in determining the wavelength at which net fiber dispersion is zero, as we shall see.
Figure 1.3: Increase in the capacity of lightwave systems realized after 1980. Commercial systems (circles) follow research demonstrations (squares) with a few-year lag. The change in the slope after 1992 is due to the advent of WDM technology.
Figure 1.5: International undersea network of fiber-optic communication systems around 2000. (After Ref. [22]; ©2000 Academic; reprinted with permission.)
WDM and TDM

Figure 8.2: Multichannel point-to-point fiber link. Separate transmitter-receiver pairs are used to send and receive the signal at different wavelengths.

TDM (time division multiplexing)

Figure 8.26: Design of an OTDM transmitter based on optical delay lines.
Optical MUX/DEMUX components

Figure 8.10: Layout of an integrated four-channel waveguide multiplexer based on Mach–Zehnder interferometers. (After Ref. [69]; ©1988 IEEE; reprinted with permission.)

Figure 8.11: Schematic of a waveguide-grating demultiplexer consisting of an array of waveguides between two free-propagation regions (FPR). (After Ref. [74]; ©1996 IEEE; reprinted with permission.)
Mach-Zehnder fiber interferometers

\[ r_2 = \frac{1}{\sqrt{2}} \]
\[ r_2' = -\frac{1}{\sqrt{2}} \]
\[ r_1 = \frac{1}{\sqrt{2}} \]

(a)

\[ \lambda_1, \lambda_2 \]
\[ \text{Path 1} \]
\[ \text{Delay} \]
\[ \text{Path 2} \]
\[ \text{Output 1} \]
\[ \lambda_1, \lambda_2 \]
\[ \text{Input 1} \]
\[ \lambda_1, \lambda_2 \]
\[ \text{Output 2} \]
\[ \text{FC1} \]
\[ \text{FC2} \]

(b)