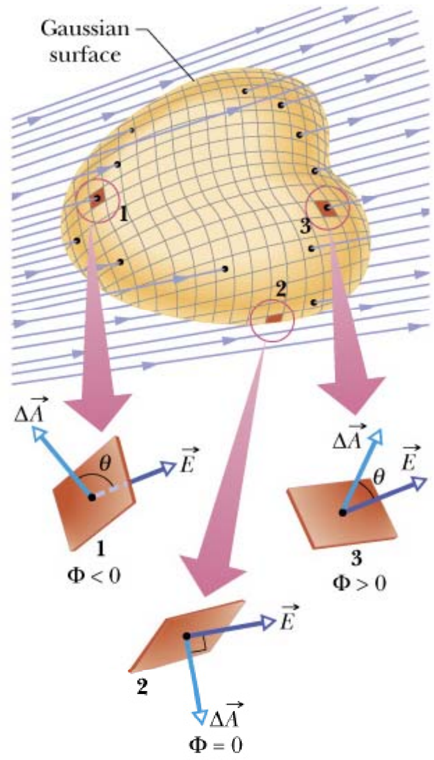
23. 가우스 법칙 (Gauss' law)

- Gauss' Law: Motivation & Definition $\oint \vec{E}\cdot d\vec{A} \equiv \Phi = \frac{q_{encl}}{\varepsilon_o}$
- Coulomb's Law as a consequence of Gauss' Law
- Charges on Conductors:
  - Where are they?
- Applications of Gauss' Law
  - Uniform Charged Sphere
  - Infinite Line of Charge
  - Infinite Sheet of Charge
  - Two infinite sheets of charge



지난 시간에 ...

전기장 (Electric field)

$$\left|\vec{E}\right| = \frac{\left|\vec{F}\right|}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{|q|}{r^2} \qquad \vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

전기 쌍극자 (Electric dipole) $\vec{p} = q\vec{d}$

$$E \approx \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\left(\frac{p}{z^3}\right)$$

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad U = -\vec{p} \cdot \vec{E}$$

연속전하의 전기장

$$\vec{E} = k_e \int \frac{\rho dV}{r^2}\hat{r} \qquad \vec{E} = k_e \int \frac{\sigma dS}{r^2}\hat{r} \qquad \vec{E} = k_e \int \frac{\lambda dl}{r^2}\hat{r}$$

Fundamental Law of Electrostatics (정전기학)

- **Coulomb's Law**

  Force between two point charges

$$\left|\vec{F}\right| = \frac{q_0}{4\pi\varepsilon_o}\frac{|q|}{r^2} = q_0\left|\vec{E}\right|$$

OR

- **Gauss' Law**

  Relationship between Electric Fields and charges

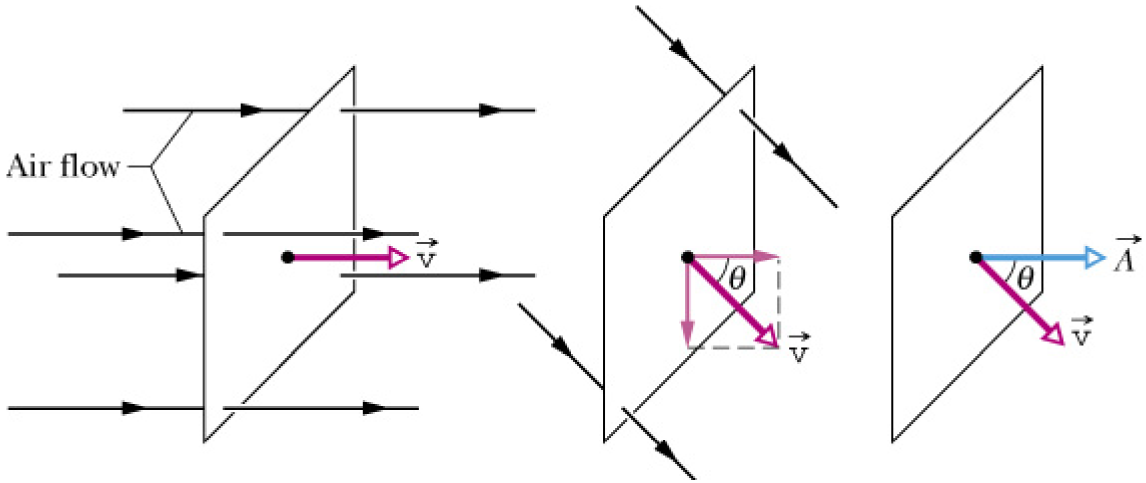
$$\oint \vec{E}\cdot d\vec{A} \equiv \Phi = \frac{q_{encl}}{\varepsilon_o}$$

23-2. 다발 (flux)

다발 (flux)

Φ = (면에 수직인 속도성분)×(면의 넓이)

$$\Phi \equiv (v\cos\theta)A = \vec{v} \bullet \vec{A}$$



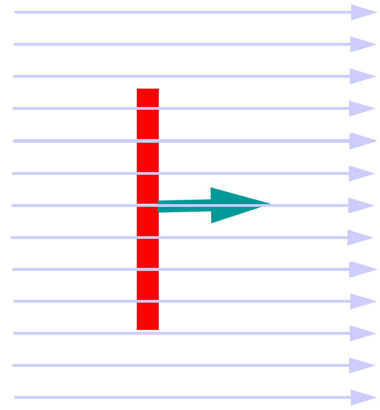
22-3. 전기장 다발 (Electric flux)

- Consider **flux through two surfaces** that "intercept different numbers of field lines"

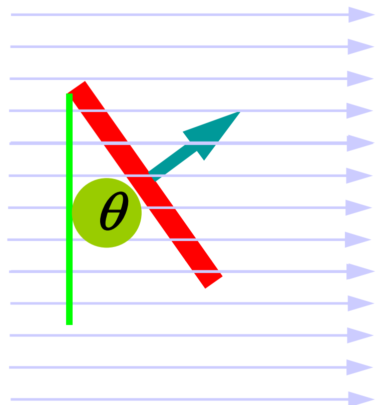
	E-field	surface area	Flux: $\vec{E}\cdot\vec{A}$
case 1	$\vec{E}=E_o\hat{y}$	w^2	$E_o w^2$
case 2	$\vec{E}=E_o\hat{y}$	w^2	$E_o w^2\cdot\cos\theta$

Case 2 is smaller!

case 1



case 2

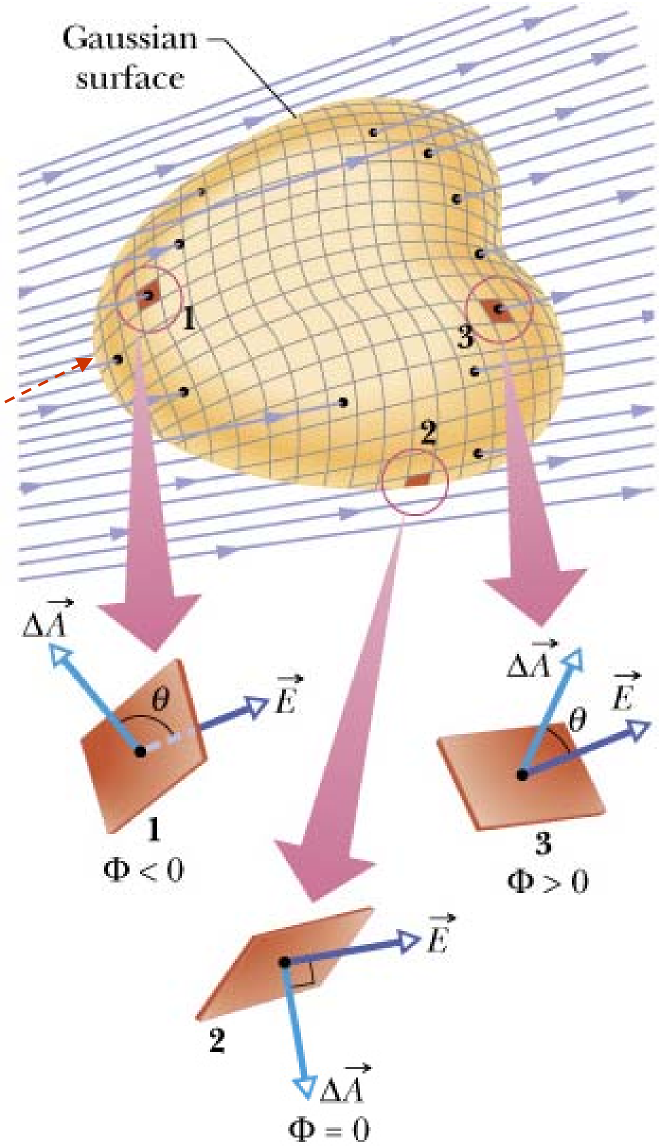


22-3. 전기장 다발 (Electric flux)

$$\Phi \equiv \lim_{\Delta A \to 0} \sum E \cdot \Delta A$$

$$= \oint_S E \cdot dA$$

Gauss 폐 곡면
(Gaussian surface)



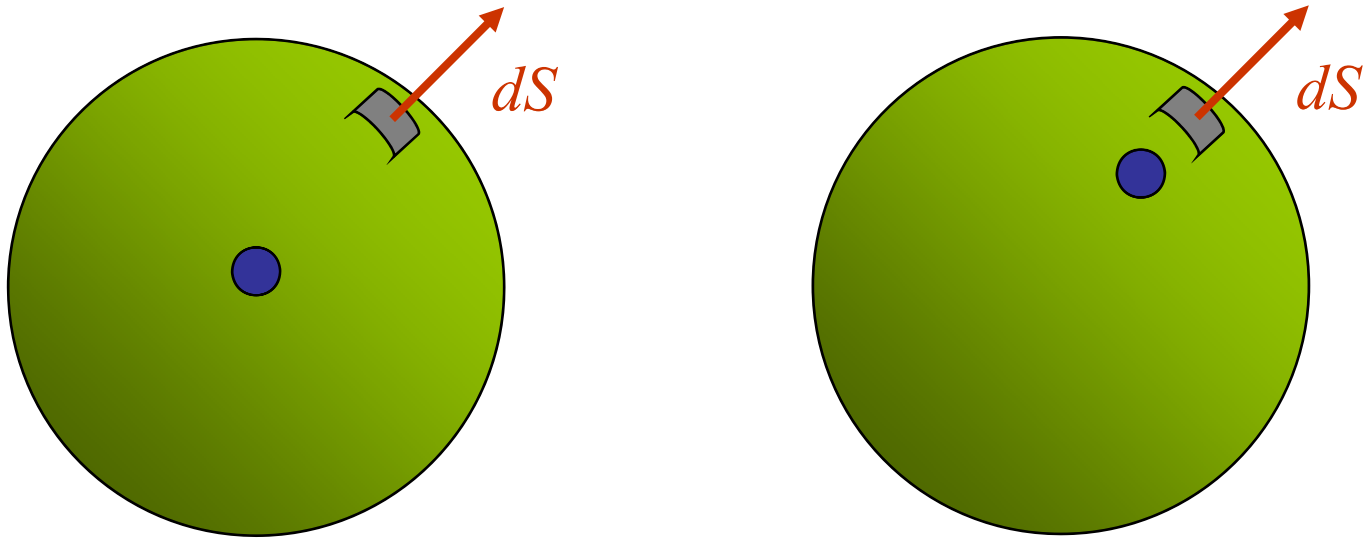
Electric Flux

$$\Phi \equiv \oint \vec{E} \cdot d\vec{A}$$

- What does this new quantity mean?

- The integral is over a CLOSED SURFACE
- Since $\vec{E} \bullet d\vec{A}$ is a SCALAR product, the electric flux is a SCALAR quantity
- The integration vector $d\vec{A}$ is normal to the surface and points OUT of the surface. $\vec{E} \bullet d\vec{A}$ is interpreted as the component of E which is NORMAL to the SURFACE
- Therefore, the electric flux through a closed surface is the sum of the normal components of the electric field all over the surface.
- The sign matters!!
  Pay attention to the direction of the normal component as it penetrates the surface… is it "out of" or "into" the surface?
- "Out of" is "+" "into" is "-"

질문

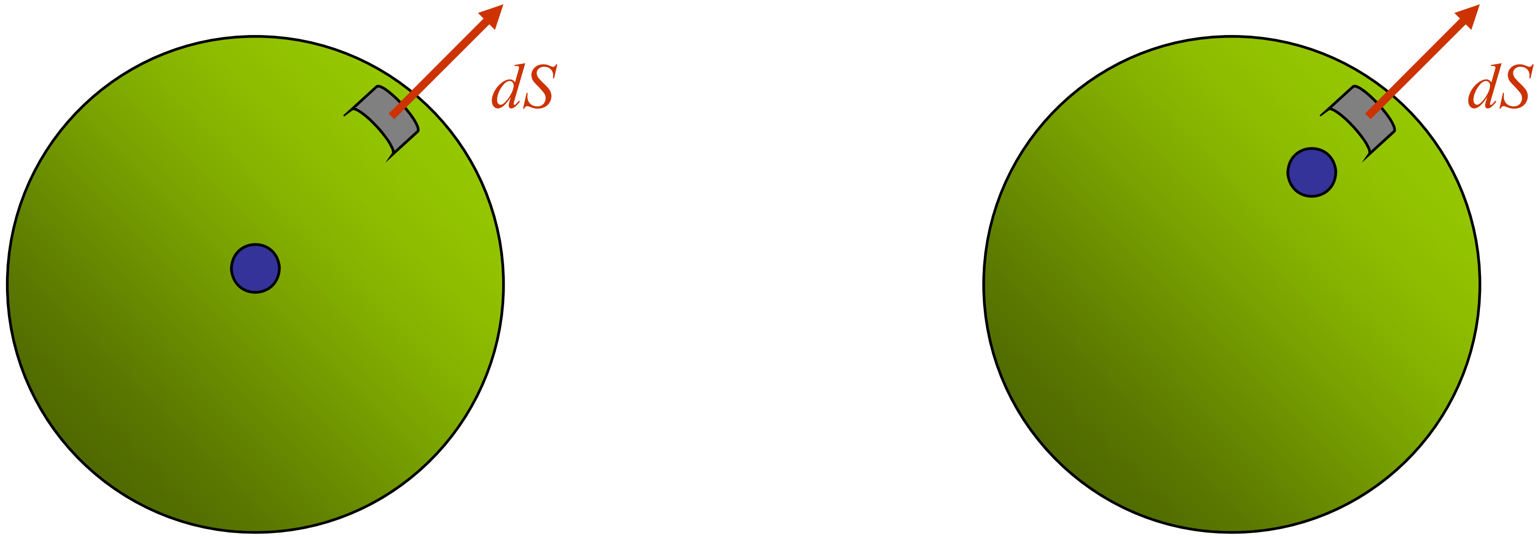


A positive charge is contained inside a spherical shell.
How does the electric flux $d\Phi_E$ through the surface element dS change when the charge is moved from position 1 to position 2?

a) $d\Phi_E$ increases

b) $d\Phi_E$ decreases

c) $d\Phi_E$ doesn't change



A positive charge is contained inside a spherical shell.

How does the flux Φ_E through the entire surface change when the charge is moved from position 1 to position 2?

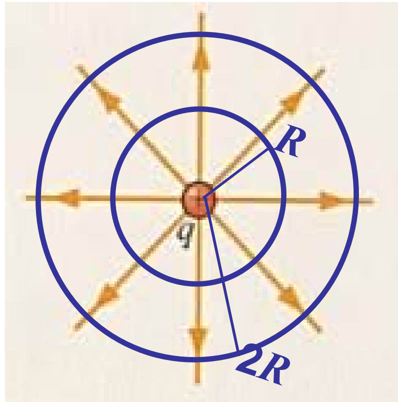
a) Φ_E increases

b) Φ_E decreases

c) Φ_E doesn't change

질문

- **Consider 2 spheres (of radius R and $2R$) drawn around a single charge as shown.**
  - **Which of the following statements about the net electric flux through the 2 surfaces (Φ_{2R} and Φ_R) is true?**



(a) $\Phi_R < \Phi_{2R}$ (b) $\Phi_R = \Phi_{2R}$ (c) $\Phi_R > \Phi_{2R}$

- **Look at the lines going out through each circle -- each circle has the same number of lines.**
- **The electric field is different at the two surfaces, because E is proportional to $1 / r^2$, but the surface areas are also different. The surface area of a sphere is proportional to r^2.**
- **Since flux = $\oint_S \vec{E} \cdot d\vec{A}$, the r^2 and $1/r^2$ terms will cancel, and the two circles have the same flux!**

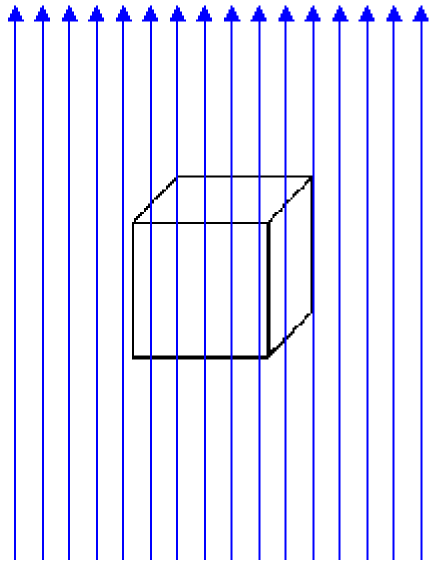
질문

A cube is placed in a uniform electric field. Find the flux through the bottom surface of the cube.

a) $\Phi_{bottom} < 0$

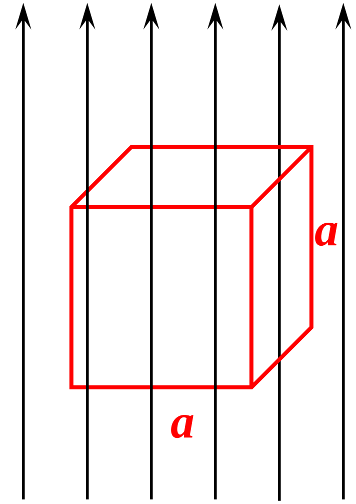
b) $\Phi_{bottom} = 0$

c) $\Phi_{bottom} > 0$



질문

Imagine a cube of side *a* positioned in a region of constant electric field as shown.
Which of the following statements about the net electric flux Φ_E through the surface of this cube is true?



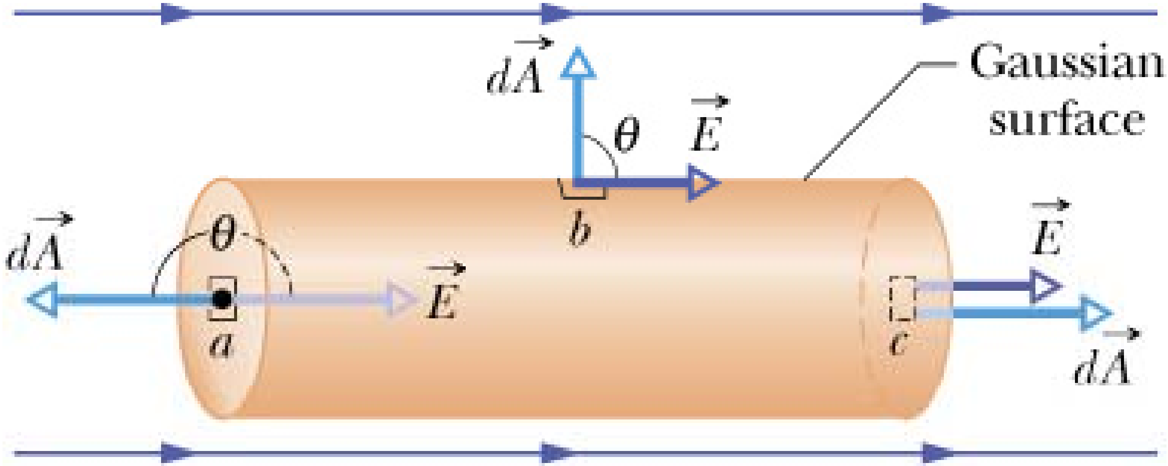
(a) $\Phi_E = 0$ **(b)** $\Phi_E \propto 2a^2$ **(c)** $\Phi_E \propto 6a^2$

- **The electric flux through the surface is defined by:** $\Phi \equiv \oint \vec{E} \cdot d\vec{A}$
- $\oint \vec{E} \cdot d\vec{A}$ **is ZERO on the four sides that are parallel to the electric field.**
  $\oint \vec{E} \cdot d\vec{A}$ **on the bottom face is negative. (dS is out; E is in)**
  $\oint \vec{E} \cdot d\vec{A}$ **on the top face is positive. (dS is out; E is out)**
- **Therefore, the total flux through the cube is:**

[redacted]

보기문제 23-1

고른 전기장 E 속에 놓인 반지름 R 인 원통꼴의 가우스 곡면에서의 전기장 플럭스 Φ의 값?



$$\Phi = \oint \mathbf{E}\cdot d\mathbf{A} = \int_a \mathbf{E}\cdot d\mathbf{A} + \int_b \mathbf{E}\cdot d\mathbf{A} + \int_c \mathbf{E}\cdot d\mathbf{A}$$

$$\int_a \mathbf{E}\cdot d\mathbf{A} = \int E(\cos 180^o)dA = -E\int dA = -EA$$

$$\int_b \mathbf{E}\cdot d\mathbf{A} = \int \mathrm{E}(\cos 90^o)\mathrm{dA} = 0$$

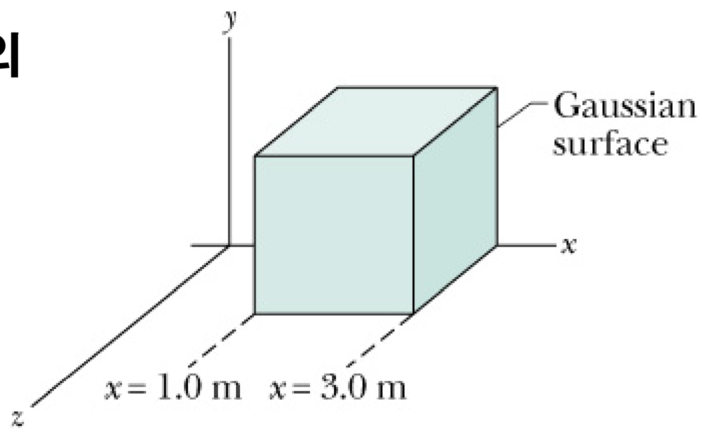
$$\int_c \mathbf{E}\cdot d\mathbf{A} = \int E(\cos 0^o)dA = EA$$

$$\Phi = -EA + 0 + EA = 0$$

보기문제 23-2

그림과 같은 정육면체의 왼쪽, 오른쪽, 위쪽 면에서의 전기장 플럭스? 단, 전기장은

$E = (3.0\,x)\,i + (4.0)\,j \quad (N/C).$



1) 왼쪽 면

$$dA = -(dydz)\,i,$$

$$E = (3.0x\,i + 4.0\,j)_{x=1.0} = (3.0)i + (4.0)j$$

$$\Phi = \int E \cdot dA = \int_{z=0}^{z=2}\int_{y=0}^{y=2} (3.0i + 4.0j)\cdot(-dydz\ i)$$

$$= -3.0\int_{z=0}^{z=2}\int_{y=0}^{y=2} dydz = -12.0\ (N \cdot m^2/C)$$

2) 오른쪽 면

$$dA = dydz\,i, \quad E = (3.0x\ i + 4.0\ j)_{x=3.0} = (9.0)i + (4.0)j$$

$$\Phi_{오른쪽} = -3\times\Phi_{왼쪽} = 36\ (N \cdot m^2/C)$$

3) 위쪽 면

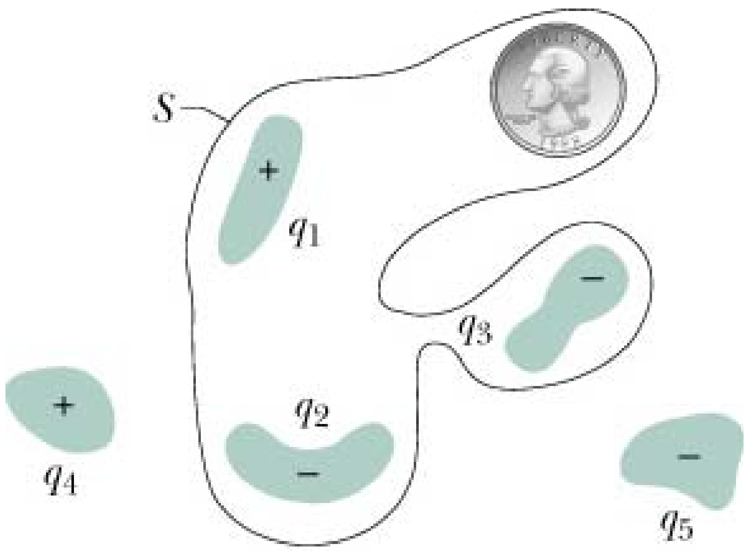
$$dA = dzdx\,j, \quad E = (3.0x\ i + 4.0\ j)_{y=2.0} = (6.0)i + (4.0)j$$

$$\Phi_{위쪽} = \int_{x=1}^{x=3}\int_{z=0}^{z=2}(3.0x\ i + 4.0\ j)\cdot(dzdx\ j) = 16.0\ (N \cdot m^2/C)$$

23-4. Gauss 법칙

$$\oint \vec{E} \cdot d\vec{A} \equiv \Phi = \frac{q_{encl}}{\varepsilon_o}$$

OR, $$\varepsilon_o \Phi = q_{encl}$$



The net electric flux through any closed surface is proportional to the charge enclosed by that surface.

It is very useful in finding *E*
when the physical situation exhibits massive SYMMETRY.

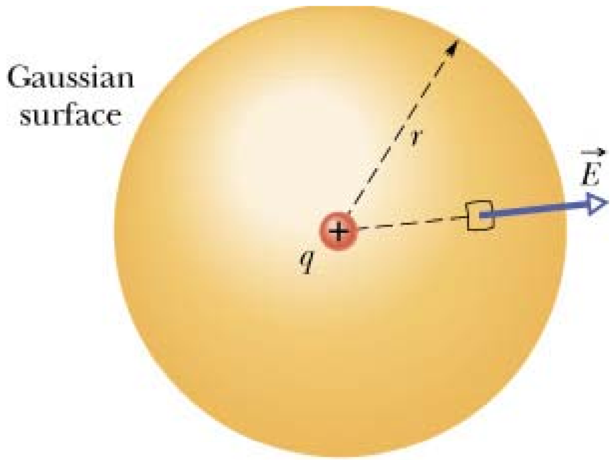
23-5. Gauss 법칙과 Coulomb 법칙

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_o}$$

가우스 법칙에서 쿨롱 법칙 끌어내기

점 전하 ***q*** 가 원점에 있을 때
반지름 ***r*** 인 공 표면에서의 전기장?

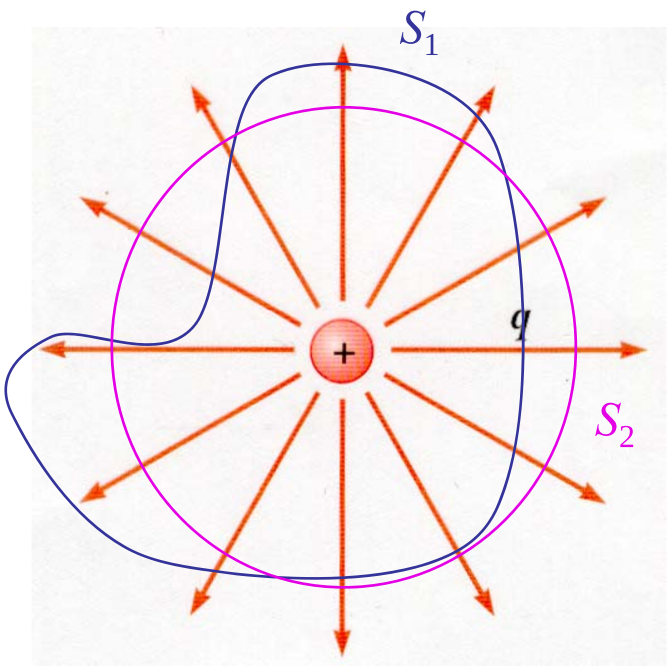
구 대칭성 때문에 공표면 어디에서나
전기장의 크기는 같고, 방향은 밖으로 나아가는 반지름 방향



$$\oint_S \mathbf{E} \cdot d\,\mathbf{A} = E \oint_S dA = E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(쿨롱 법칙)}$$

Gauss 폐곡면의 선택



$$\Phi_C = \oint_{S_1} \vec{E}\cdot d\vec{A} = \oint_{S_2} \vec{E}\cdot d\vec{A}$$

$$= k_e \frac{q}{r^2}\cdot 4\pi r^2 = 4\pi k_e q$$

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

$$\Phi_C = \frac{q}{\varepsilon_0}$$

For any empirical surface, Φ_C is the same.

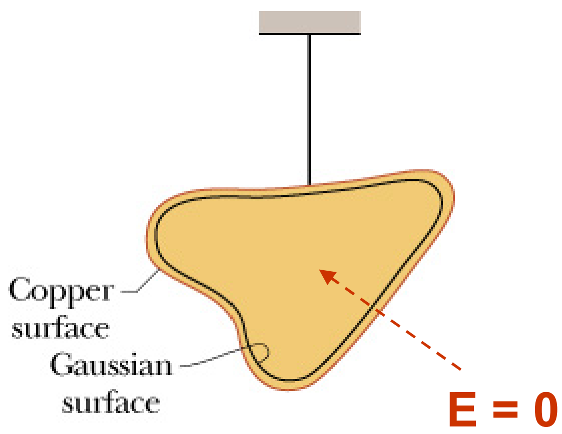
i.e. Φ_C is independent of the surface contour.

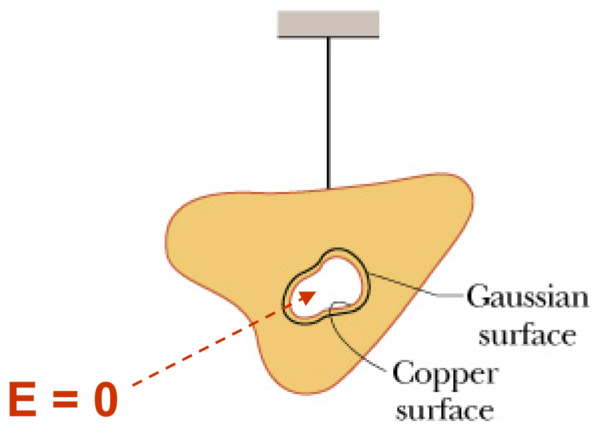
23-6 전하를 띤 고립된 도체

고립된 도체에 들어 있는 잉여전하 (excess charge)는
도체의 표면에 퍼져 있게 된다.

- 서로 밀어내는 정전기력

- 도체 속에서는 전기장이 → E = 0 (왜?)



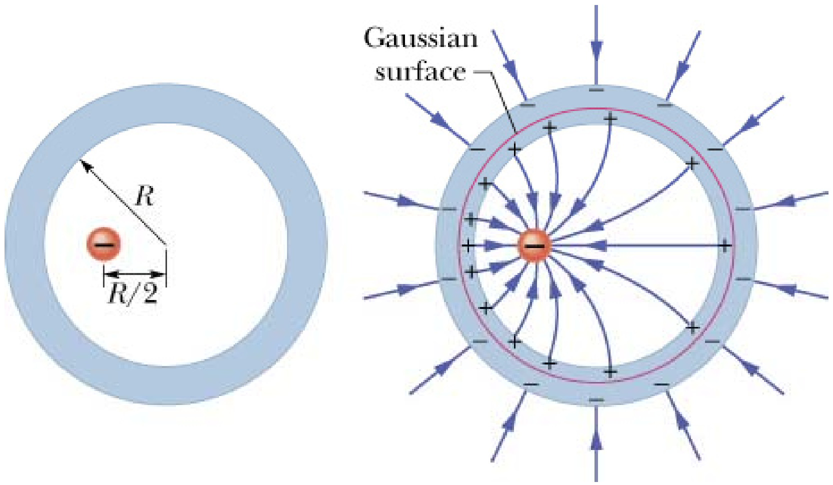


보기문제 23-4

1) 쇠공의 안팎의 면에 쌓인 전하량?

2) 안팎의 면의 전하분포?

속빈 쇠공: 속 반지름 ***R*****,** 전기적으로 중성
점전하: **- 5.0 μC,** 중심에서 ***R*****/2**인 곳



1) 쇠공의 안쪽 면의 전하량**:**

쇠 속은 전기장이 **0**

⇒ 쇠 속의 가우스 면에서도 전기장이 **0**

⇒ 가우스면 전체의 플럭스도 **0**

⇒ 가우스면 속의 알속전하는 **0**

⇒ 쇠공 안쪽 면에 **+5.0 μC** 전하 분포

2) 쇠공의 바깥 면의 전하량**:**

쇠공은 전기적으로 중성**,** 전하는 보존

⇒ 쇠공 바깥 면에 **- 5.0 μC** 전하 분포

▸ **전하분포는 공 안쪽은 고르지 않으나 바깥쪽은 고름**

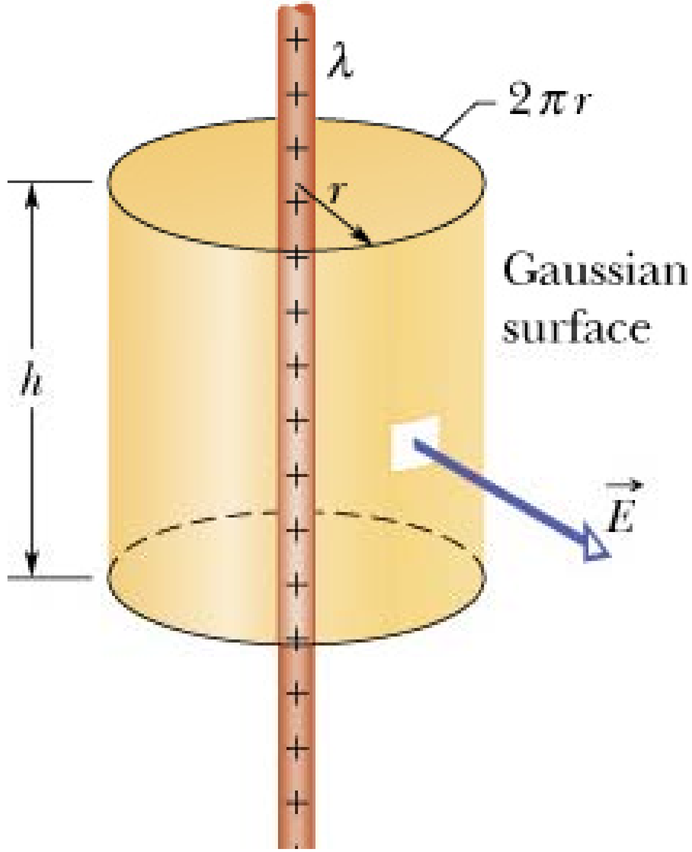
23-7 Gauss 법칙의 적용: 선전하 - 원통대칭

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Phi = EA\cos\theta = E(2\pi rh)$$

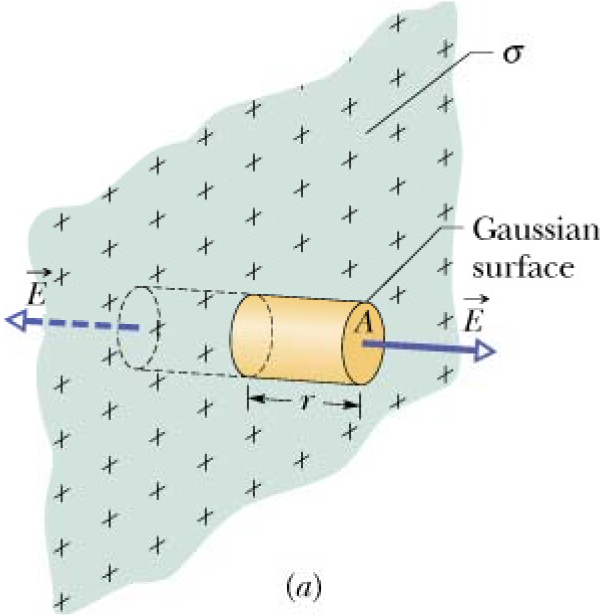
$$q_{속} = \lambda h$$

$$\therefore E = \frac{1}{2\pi\varepsilon_0}\left(\frac{\lambda}{r}\right)$$

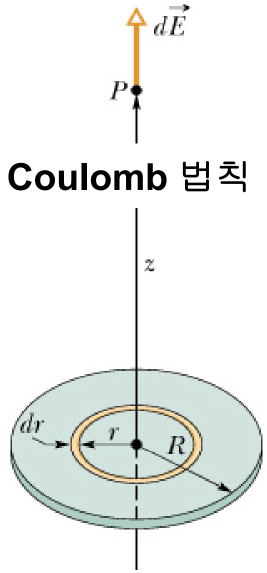


23-8 Gauss 법칙의 적용: 면전하 - 원통대칭

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_o}$$



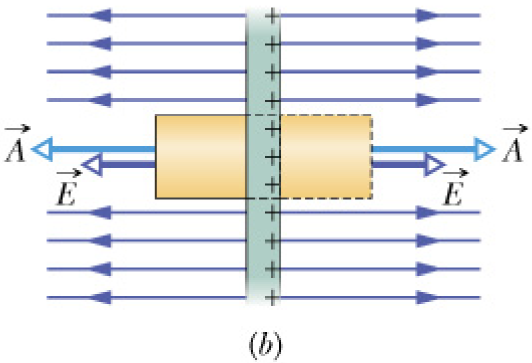
(a)



$$\Phi = EA + EA = 2EA$$

$$q_{속} = \sigma A$$

$$\therefore E = \frac{\sigma}{2\varepsilon_0}$$



(b)

$$\left|\vec{E}\right| = E_z = \frac{\sigma z}{4\varepsilon_o}\int_0^R \frac{2r}{\left(z^2 + r^2\right)^{3/2}} dr$$

$$= \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

If $R \rightarrow$ *infinite,*

$$E \approx \frac{\sigma}{2\varepsilon_o}$$

(면부터의 거리에 무관하게 일정)

두 도체판

- **Field outside must be zero.**

  Two ways to see:

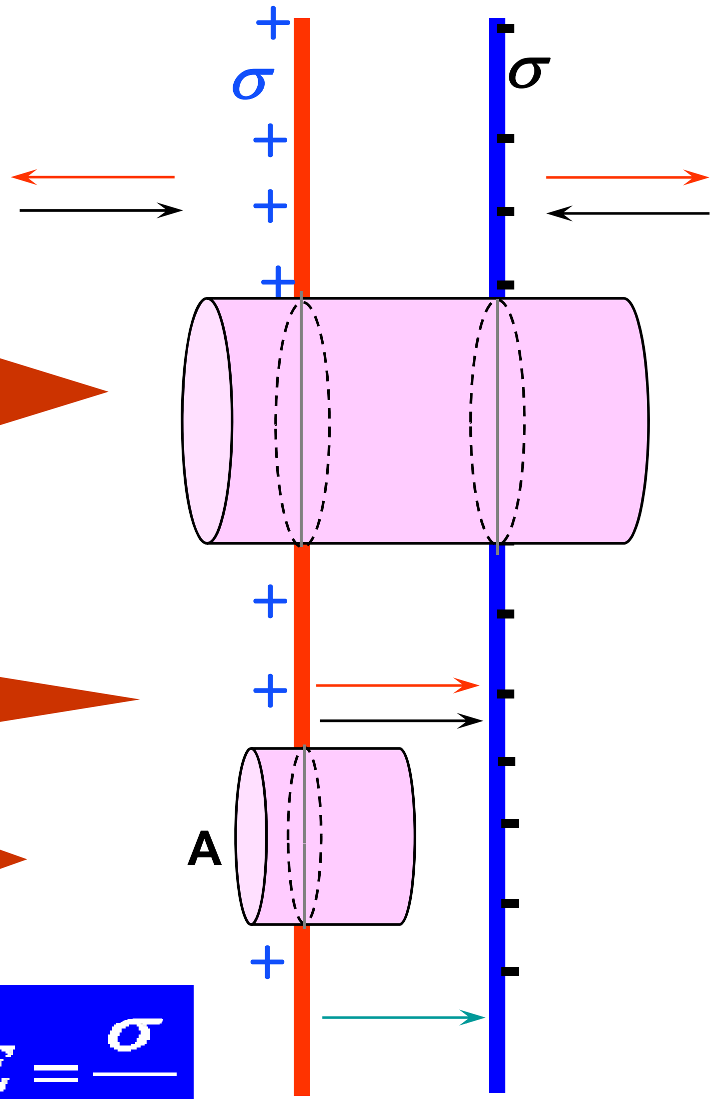
  – **Superposition**

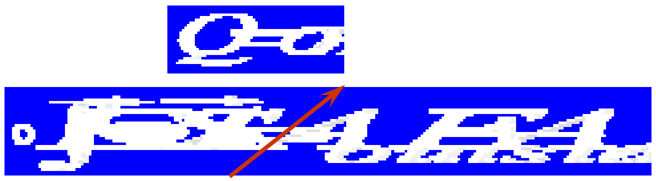
  – **Gaussian surface encloses zero charge**

- **Field inside is NOT zero:**

  – **Superposition**

  – **Gaussian surface encloses non-zero charge**





$$E = \frac{\sigma}{\varepsilon_0}$$

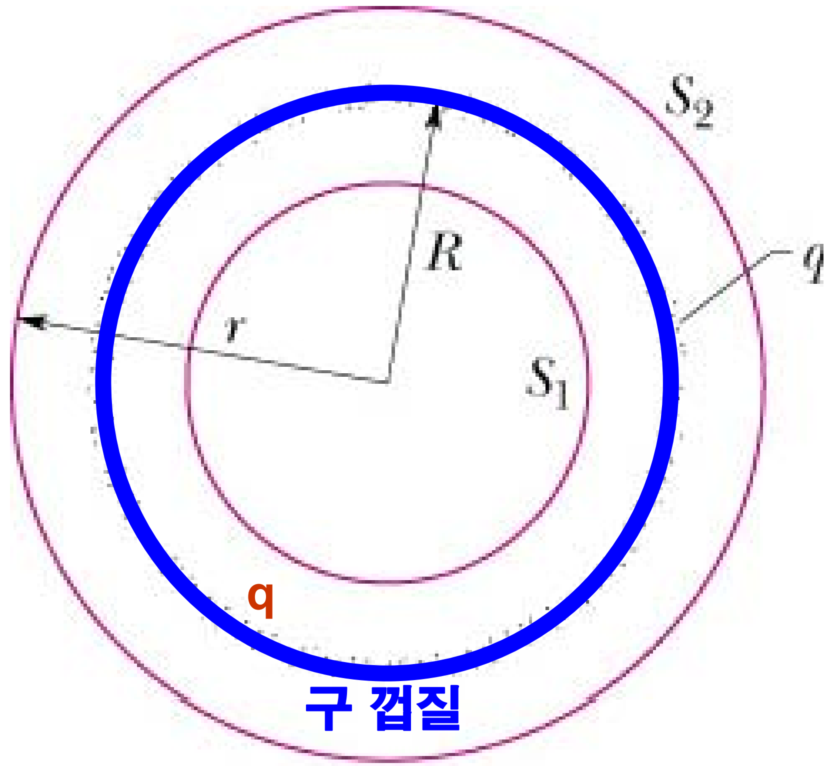
23-9 Gauss 법칙의 적용: 구 껍질 - 구대칭

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_o}$$

$$\Phi = 4\pi r^2 E$$

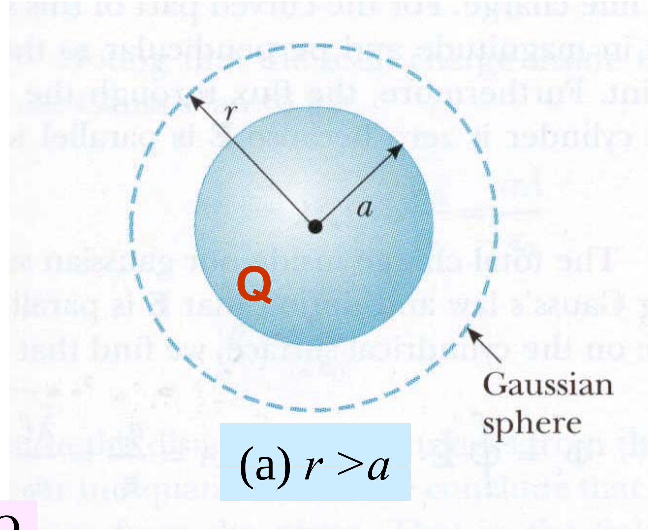
$$q_{속} = q, \quad (r > R, \text{ 공바깥}) \quad \text{(S}_2\text{)}$$
$$= 0, \quad (r < R, \text{ 공속}) \quad \text{(S}_1\text{)}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right), \quad (r > R, \text{ 공바깥})$$
$$= 0, \quad (r < R, \text{ 공속})$$



구대칭 전하분포

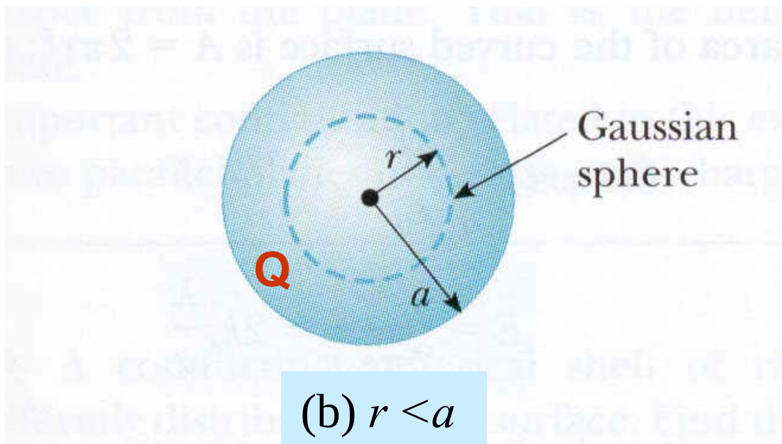
Charge density ρ : $\quad \frac{4}{3}\pi a^3 \rho = Q \quad \Rightarrow \rho = \dfrac{3Q}{4\pi a^3}$



(a) $r > a$

(a) $r > a$ $\quad \Phi_C = \oint \vec{E} \cdot d\vec{A} = \dfrac{q_{inc}}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0}$

$$4\pi r^2 \cdot E = \frac{Q}{\varepsilon_0} \qquad \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k_e \frac{Q}{r^2} \qquad \textit{for } r > a$$



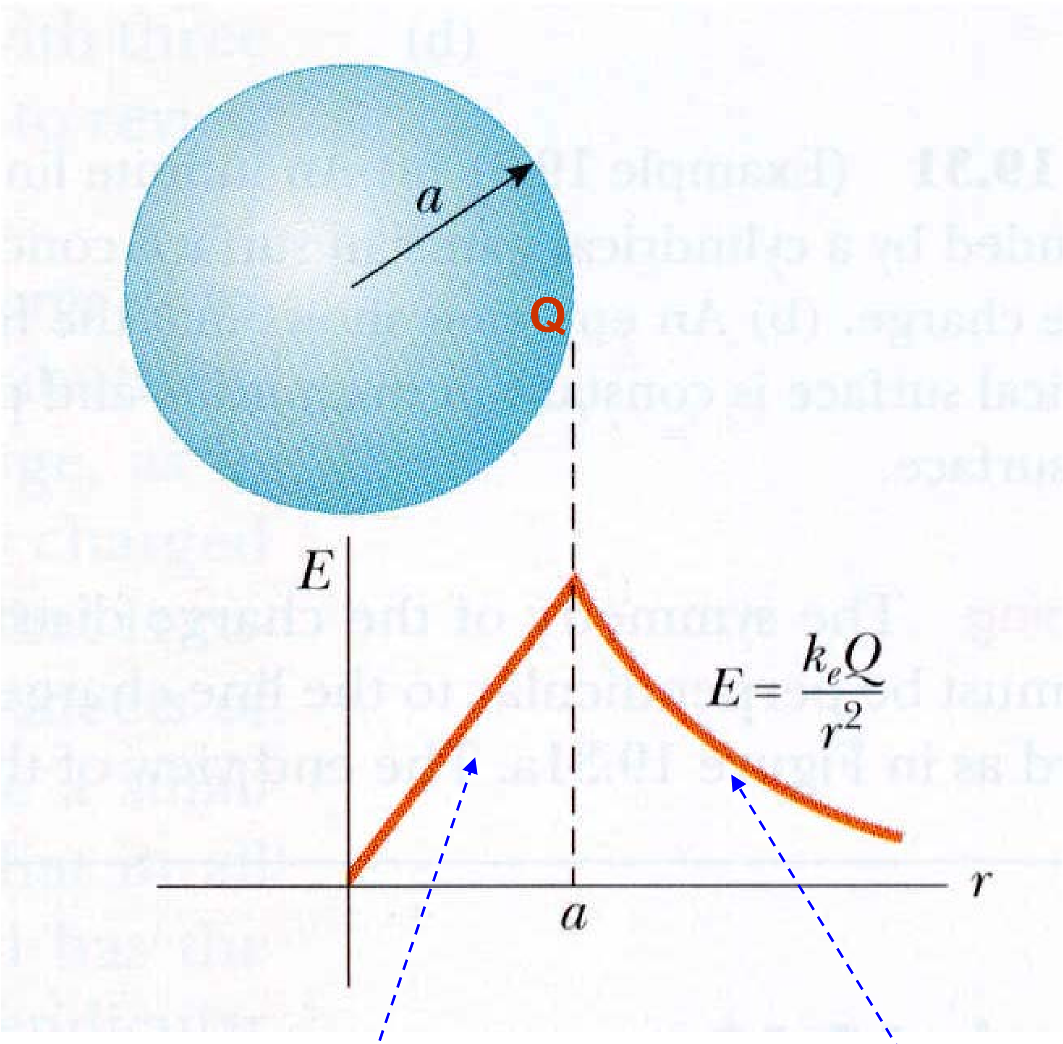
(b) $r < a$

(b) $r < a$ $\quad \Phi_C = \oint \vec{E} \cdot d\vec{A} = \dfrac{q_{inc}}{\varepsilon_0}$

$$= \frac{1}{\varepsilon_0}\int \rho \cdot dV$$

$$= \frac{\rho}{\varepsilon_0}\frac{4\pi}{3} r^3$$

$$\left(\rho = \frac{3Q}{4\pi a^3}\right) \quad 4\pi r^2 E = \frac{Q}{\varepsilon_0}\frac{r^3}{a^3} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r}{a^3} = k_e \frac{Qr}{a^3} \qquad \textit{for } r < a$$



for r < a

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r}{a^3} = k_e\frac{Qr}{a^3}$$

for r > a

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k_e\frac{Q}{r^2}$$

23. Summary

Electric Flux

$$\Phi \equiv \oint \vec{E} \cdot d\vec{A}$$

Gauss 법칙

$$\oint_S \vec{E} \cdot d\vec{A} \equiv \Phi = \frac{q_{encl}}{\varepsilon_o}$$

OR,

$$\varepsilon_o \Phi = q_{encl}$$