Canterbury Christ Church
University College

Third Year Science

Imaging

Holography

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Introduction

◆ Photograph

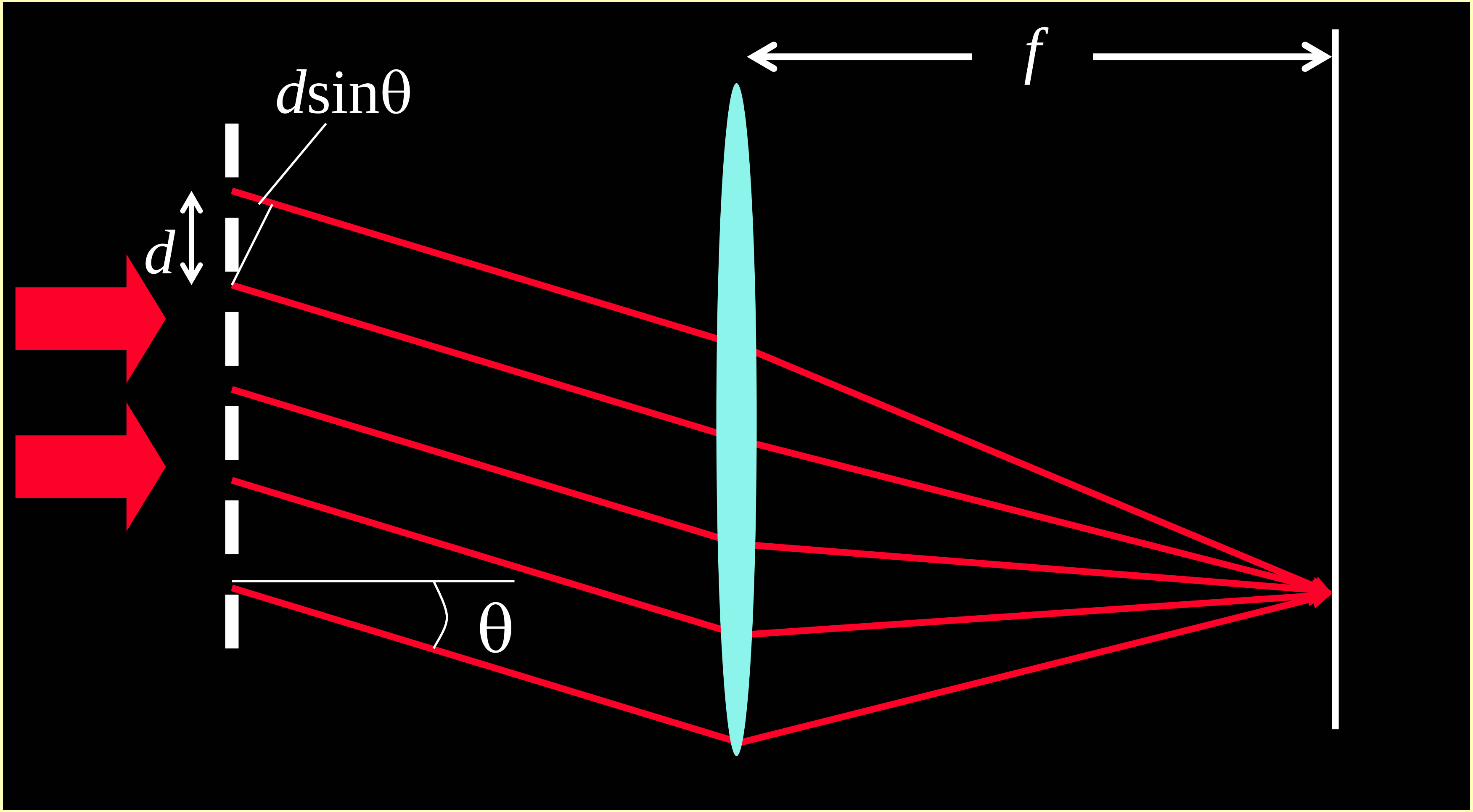
- **Image intensity distribution of light .**
  - **Does not record direction.**
  - **Two-dimensional image.**

◆ Hologram

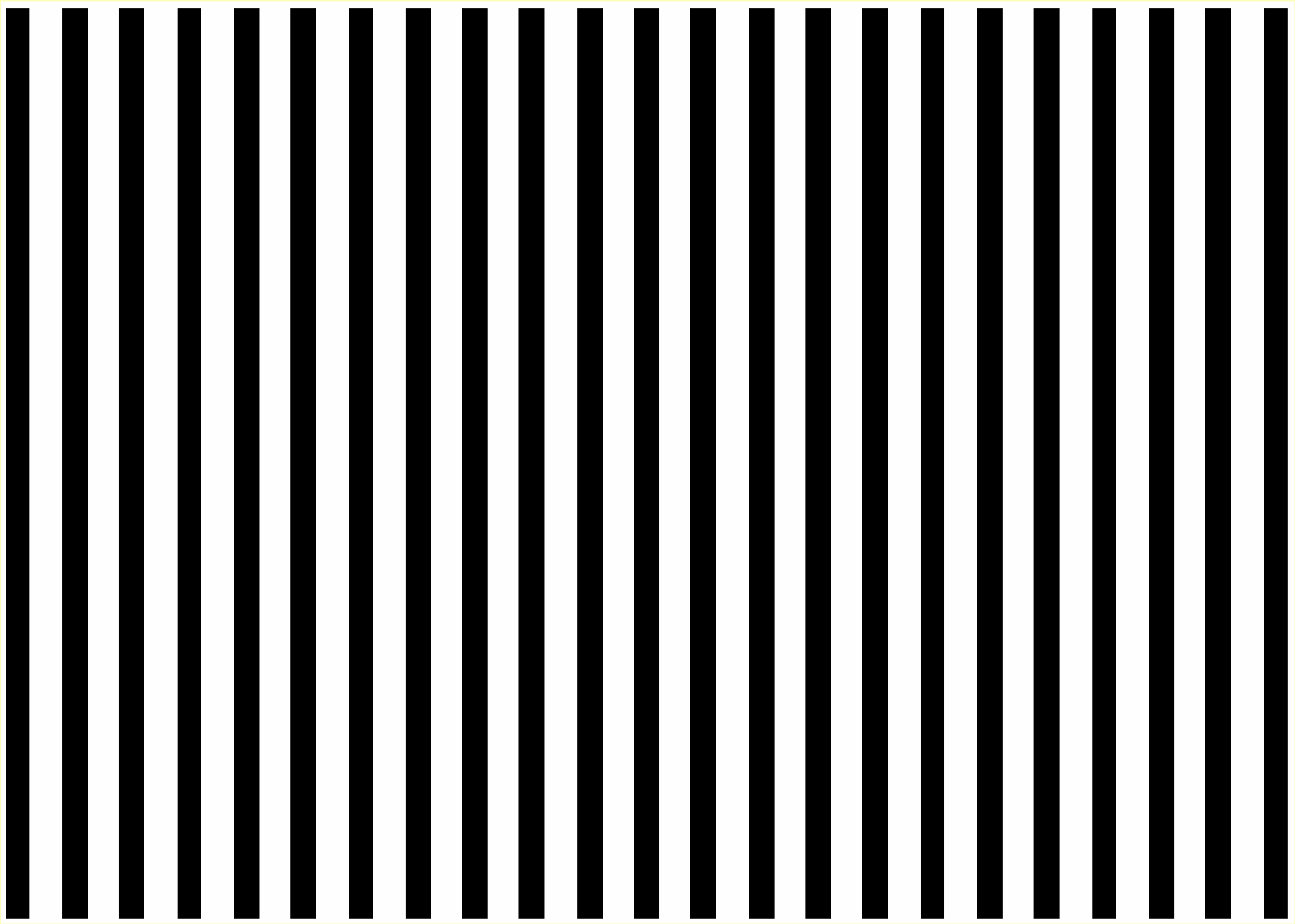
- **Records intensity & direction of light.**
  - **Information in interference pattern.**
  - **Reconstruct image by passing original light through hologram.**
  - **Need laser so that light interferes.**

Diffraction Grating

- **Series of very closely spaced slits**



Diffraction Grating



Diffraction Grating

- ***N* narrow slits**
  - **Width = *a***
  - **Separation = *d***
- **Intensity diffraction pattern is**

$$I \propto A^2 \frac{\sin^2\left(Nkd\sin\theta\right)}{\sin^2\left(kd\sin\theta\right)}$$

  - **A^2 is diffraction pattern from each slit**

$$A^2 \propto \frac{\sin^2\left(\tfrac{1}{2}ka\sin\theta\right)}{\left(\tfrac{1}{2}ka\right)^2}$$

Diffraction Grating

Intensity pattern

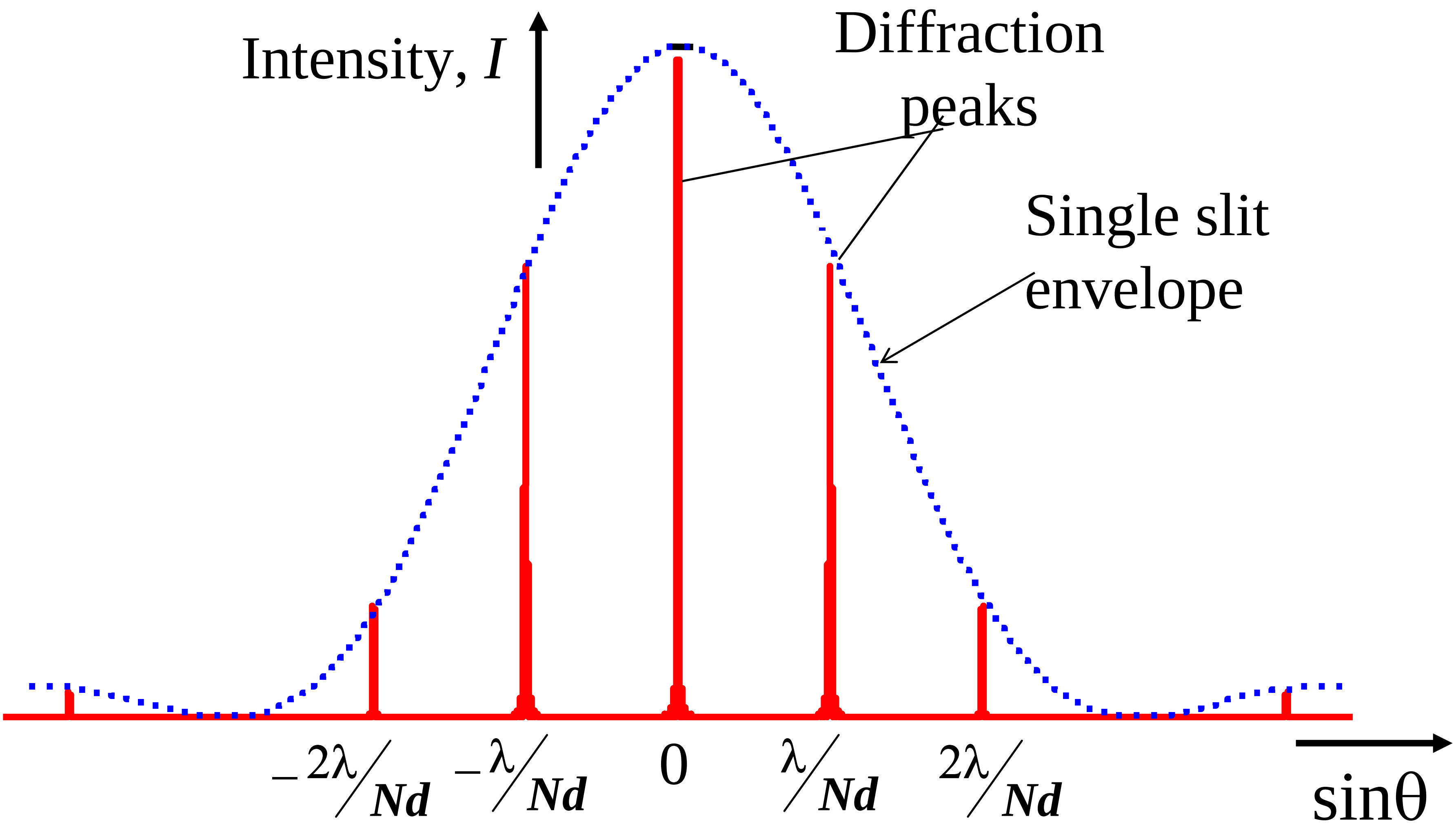
- **set of very sharp peaks**
  - **at angles given by**

$$\sin\theta = 0,\ \pm\lambda/Nd,\ \pm 2\lambda/Nd,\ \pm 3\lambda/Nd,\cdots$$

- **Amplitude of peaks**
  - **modulated by single slit pattern**

$$\frac{\sin^2\left(\tfrac{1}{2}ka\sin\theta\right)}{\left(\tfrac{1}{2}ka\right)^2}$$

Diffraction Grating

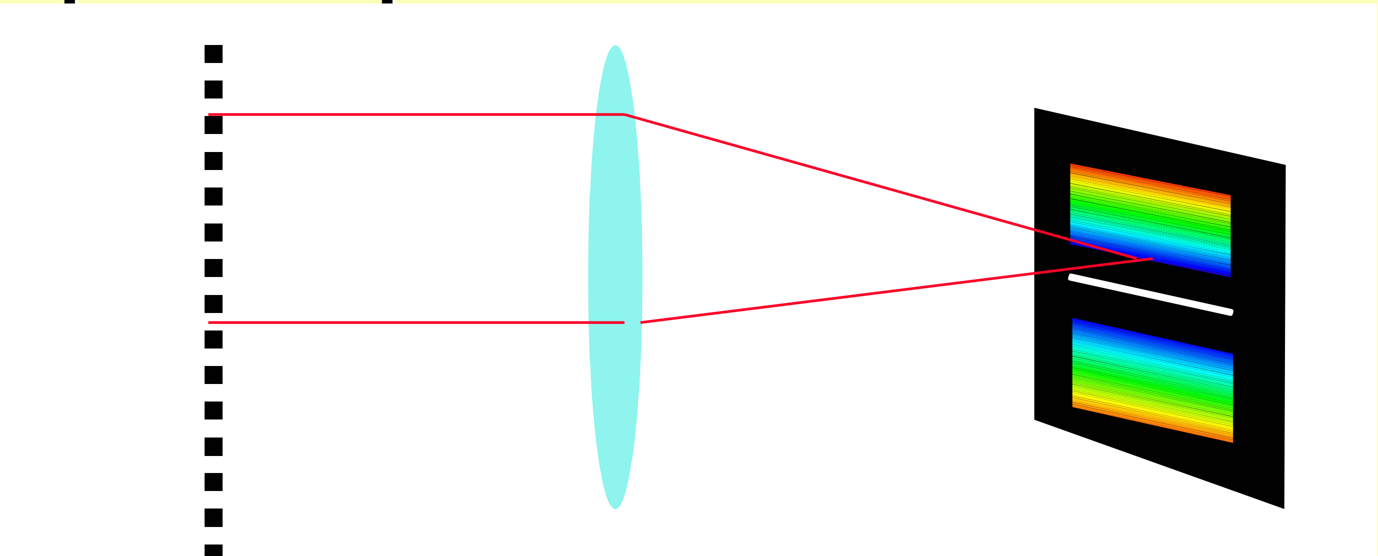


Diffraction grating

- **Can be used to display spectrum**
  - **Usually use first order peak**

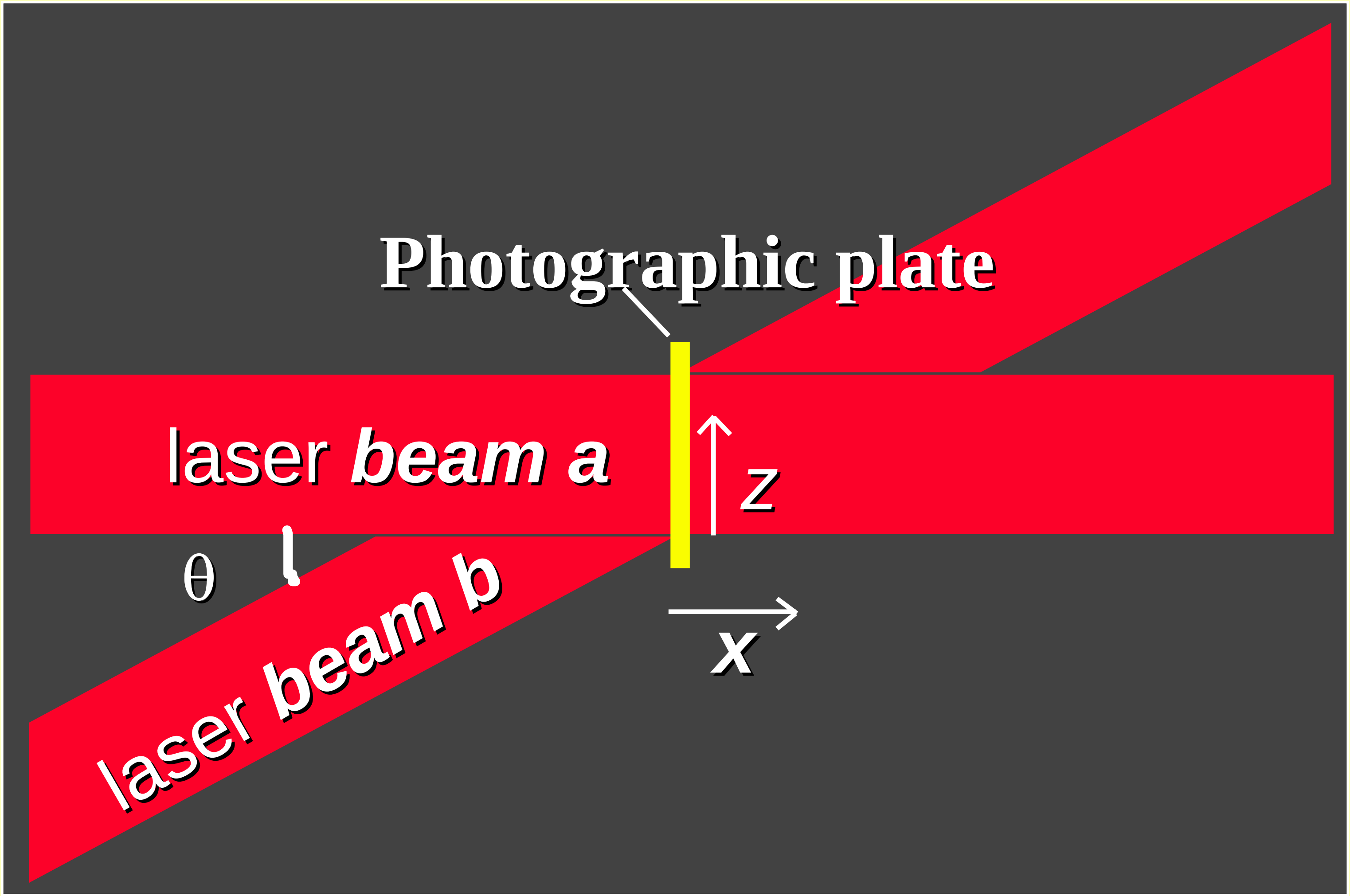
$$\sin\theta = \pm \lambda / Nd$$

  - **Since angle depends on wavelength**
    - **spectrum spread out.**

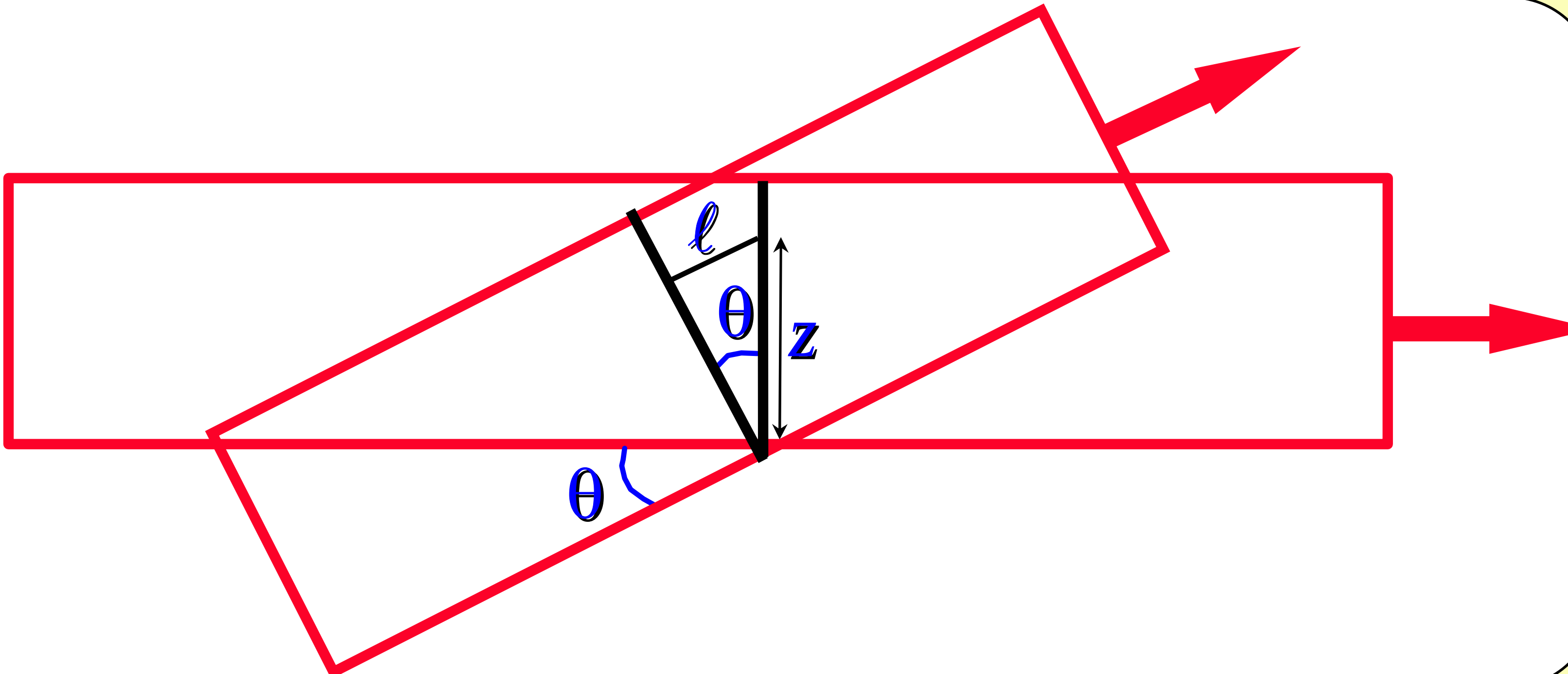


Simple Hologram

- **Two beams cross at an angle θ**



Simple Hologram



- **Extra path is** $\ell = z\sin\theta$
- **Displacements of two beams are**

$$E_a = E_o\cos\left(kx - \omega t\right)$$

$$E_b = E_o\cos\left(k\left(x + z\sin\theta\right) - \omega t\right)$$

Simple Hologram

- **Thus displacement at film is:**

$$E = E_1 + E_2 = E_o \left\{ \cos\left(kx - \omega t\right) + \cos\left(k\left[x + \ell\right] - \omega t\right) \right\}$$

$$= 2E_o \cos\left(k\left[x + \tfrac{1}{2}\ell\right] - \omega t\right) \cos\left(\tfrac{1}{2}k\ell\right)$$

  - **Using the trig identity**

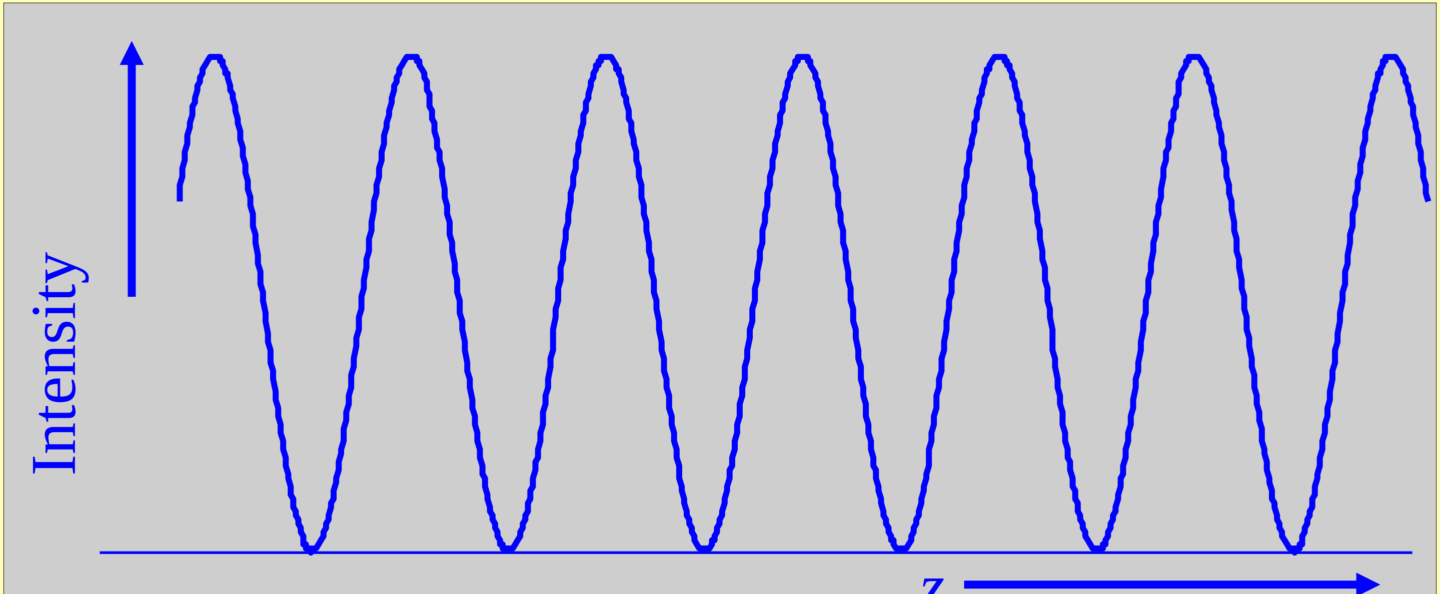
$$\cos A + \cos B = 2\cos\tfrac{1}{2}\left(A + B\right)\cos\tfrac{1}{2}\left(A - B\right)$$

  - **Amplitude varies as** $\cos\left(\tfrac{1}{2}k\ell\right)$
  - **Intensity varies as**

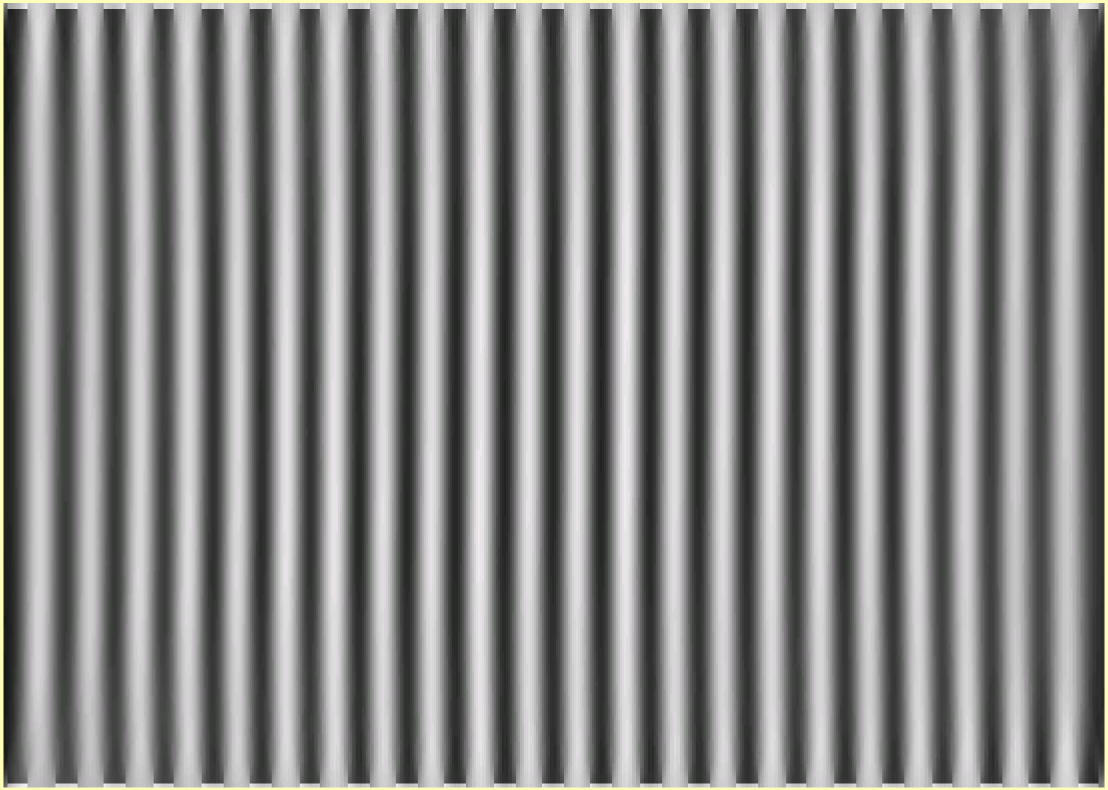
$$I \propto \cos^2\left(\tfrac{1}{2}k\ell\right) = \cos^2\left(\tfrac{1}{2}kz\sin\theta\right)$$

Simple Hologram

- **Plate is developed, and horizontal lines appear.**
- **Beam a is shone through plate**
  - **amplitude varies as** $\cos^2\left(\frac{1}{2}kz\sin\theta\right)$



Sinusoidal Diffraction Grating



Simple Hologram

- **Amplitude of beam is:**

$$E = E_o \cos^2\left(\tfrac{1}{2} kz \sin\theta\right)\cos\left(kx - \omega t\right)$$

$$= \tfrac{1}{2} E_o \left\{1 + \cos\left(kz\sin\theta\right)\right\}\cos\left(kx - \omega t\right)$$

Since $\cos^2 C = \tfrac{1}{2}\left(1 + \cos 2C\right)$

And

$$E = \tfrac{1}{2} E_0 \cos\left(kx - \omega t\right) + \tfrac{1}{2} E_o \cos\left(kz\sin\theta\right)\cos\left(kx - \omega t\right)$$

$$= \tfrac{1}{2} E_0 \cos\left(kx - \omega t\right) + \tfrac{1}{4} E_0 \cos\left(k\left(x + z\sin\theta\right) - \omega t\right)$$

$$+ \tfrac{1}{4} E_0 \cos\left(k\left(x - z\sin\theta\right) - \omega t\right)$$

since $\cos A \cos B = \tfrac{1}{2}\cos\left(A + B\right) + \tfrac{1}{2}\cos\left(A - B\right)$

Simple Hologram

- **The three parts are:**

$$\tfrac{1}{2}E_0\cos\left(kx-\omega t\right)$$

- **Beam continuing in direction of** beam a

$$\tfrac{1}{4}E_0\cos\left(k\left(x+z\sin\theta\right)-\omega t\right)$$

- **Beam in direction +θ.**

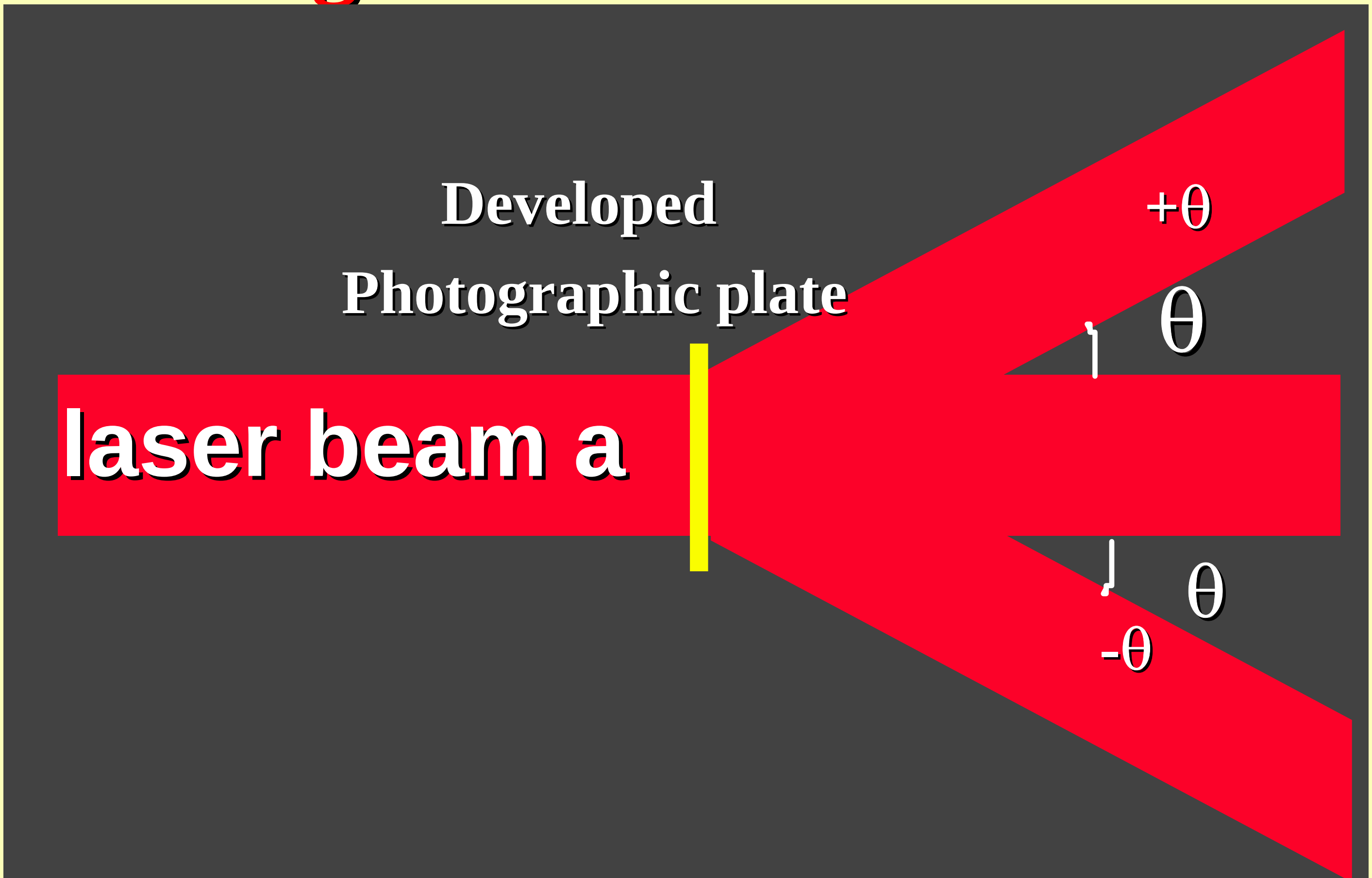
$$\tfrac{1}{4}E_0\cos\left(k\left(x-z\sin\theta\right)-\omega t\right)$$

- **Beam in direction -θ.**

- **Three beams emerge, one in direction 0, one at +θ and one at -θ.**

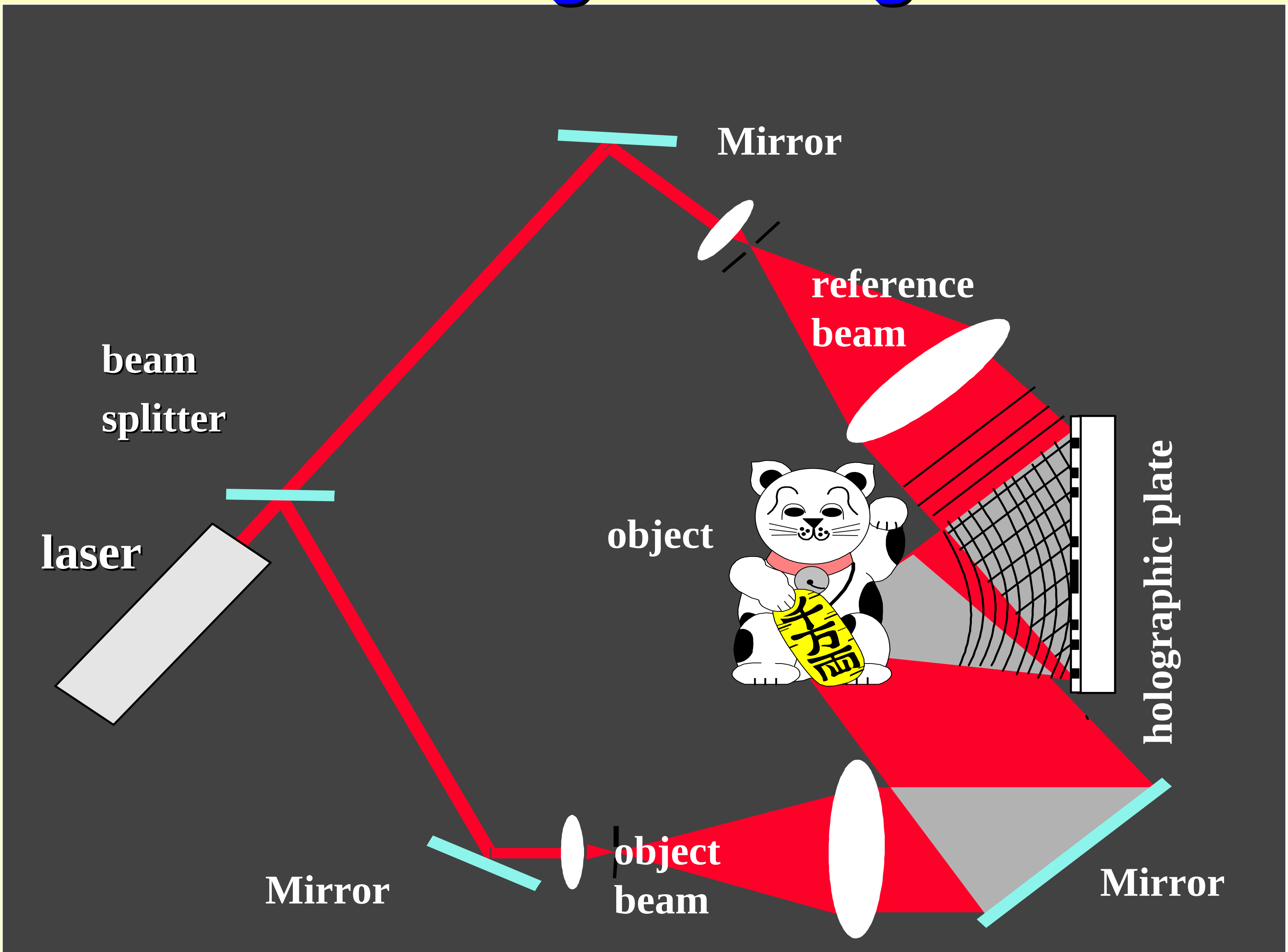
Simple Hologram

- **You will get:**



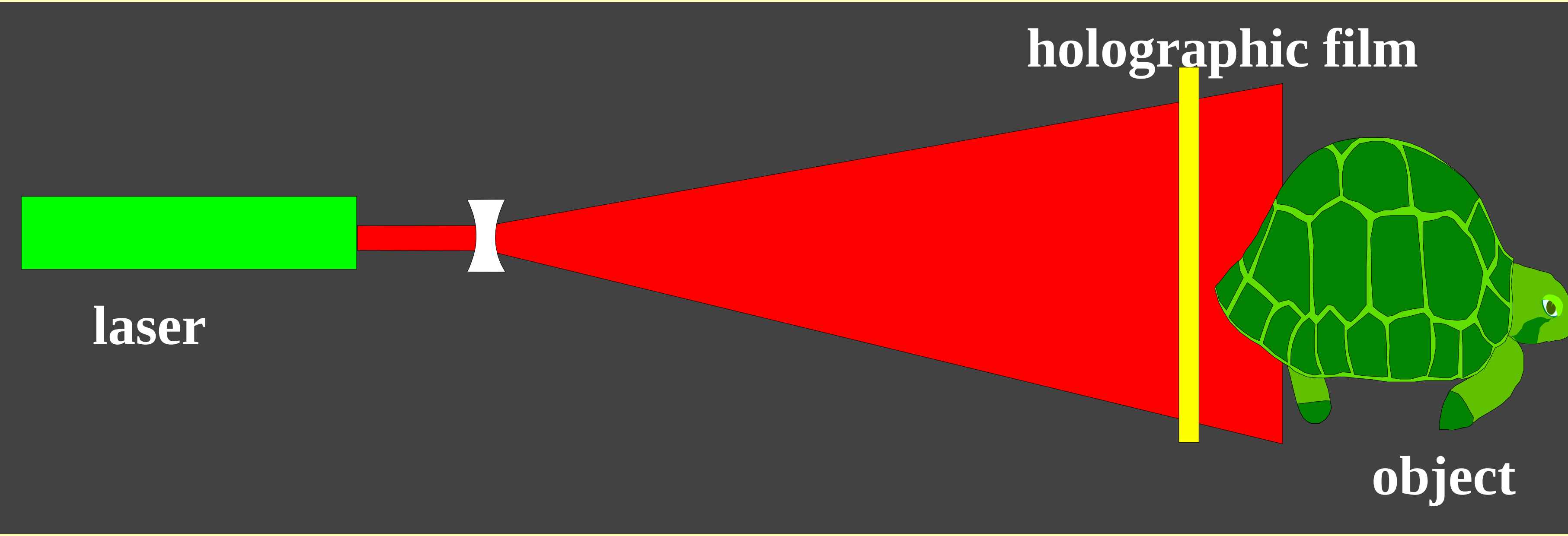
- **+θ recreation of beam b (virtual image)**
- **-θ beam is real image**

Recording a Hologram



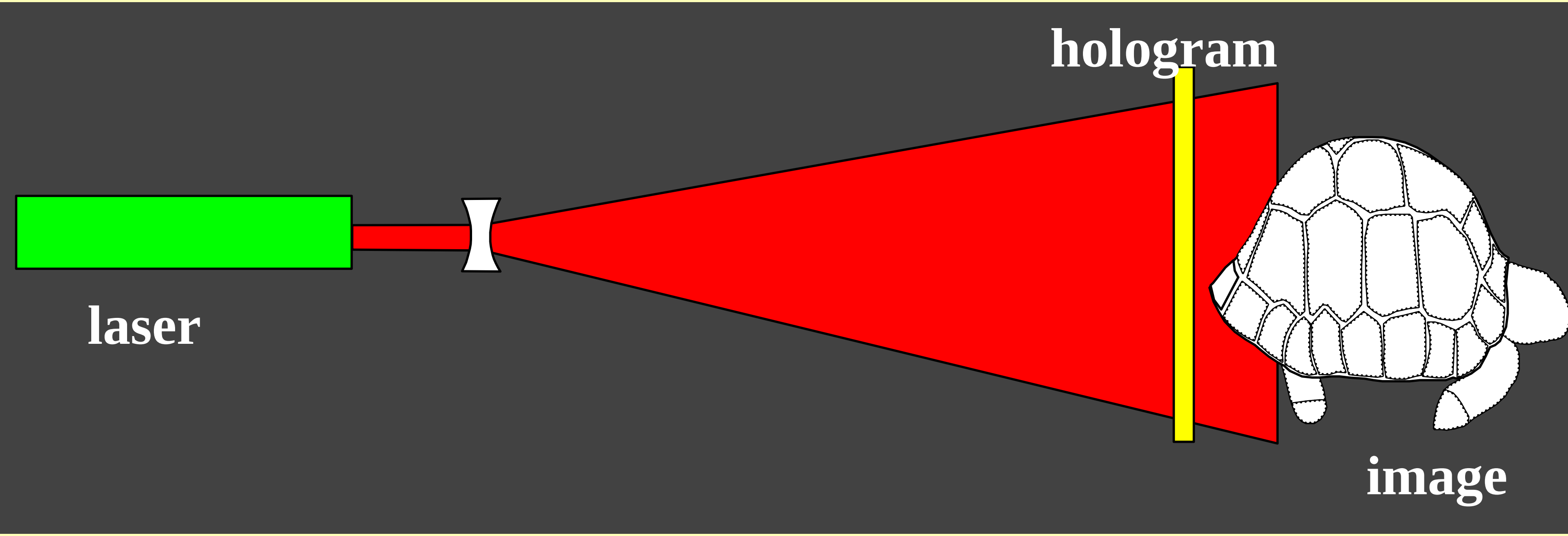
Reconstructing a hologram

In line Hologram



- **Hologram is interference pattern between direct beam and light scattered form object**

In line Hologram

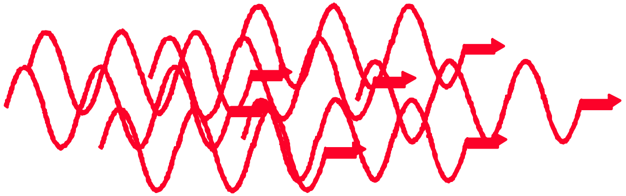


- **Hologram produced by developing film**
  - **Image seen when original beam shone on hologram**

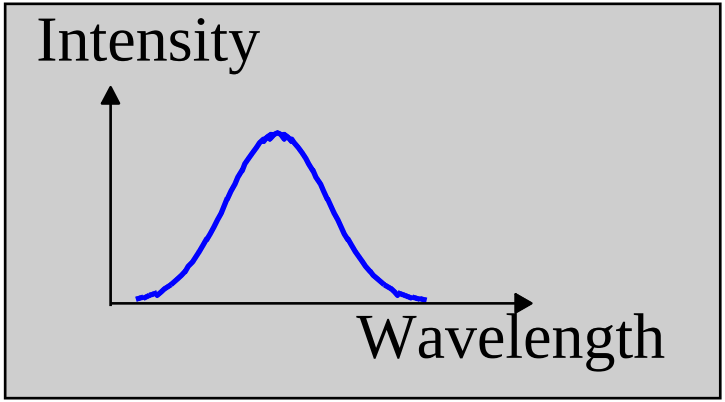
Holography

- **Many different optical arrangements.**
- **Recording requirements:**
  - Laser light source(coherent light)
  - Holographic film needs small grains.
  - Good stability (no movements during exposure)
- **Reconstruction requirements:**
  - Much less strict.
  - Some do not need laser.

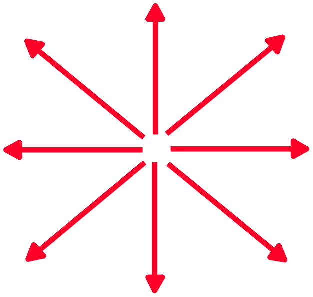
‘Conventional’ light



Short, random wave trains (Incoherent)

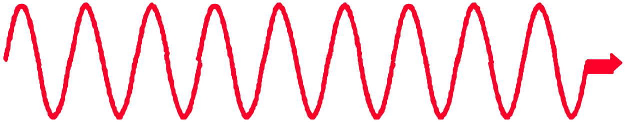


Large spread in wavelengths

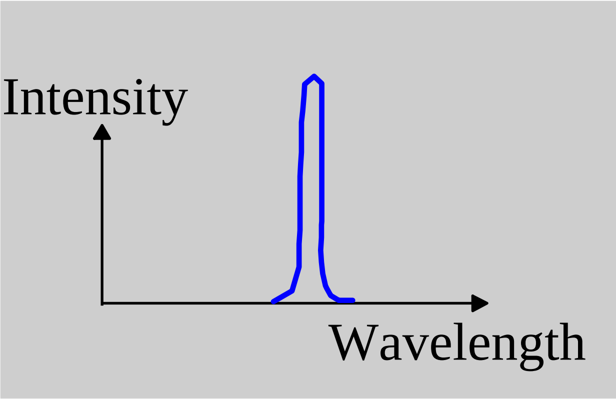


Light emerges in many directions

Laser Light

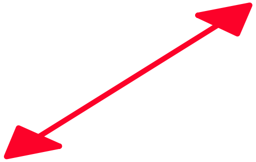


Long wave trains (Coherent)



Small spread in wavelengths

Narrow, intense light beam



Often plane polarized

Coherence

- **Interference needs fixed relationship between phases.**
  - **Conventional light sources produce short wavetrains at random intervals.**
    - **Random variation of phase**
    - **Little interference**
  - **Lasers produce long wavetrains**
    - **Phase relation between light beams fairly constant.**
    - **Good interference**

Applications of Holography

- **Artistic creations.**
- **Storing & transporting delicate images**
  - **Russian icons are shown as holograms.**
- **Holographic Interferometry.**
  - **Strain analysis of objects under stress**
  - **Used for measuring shape of objects.**
- **Data storage.**
  - **Contain large amount of visual information**

Holographic Interferometery

- **Measures strain on an object.**
- **Two methods:**
  - **Double exposure holographic interferometry.**
  - **Real time holographic interferometry.**

Holographic Interferometery

- **Double exposure holographic interferometry.**
  - **Two holograms on photographic plate.**
  - **Object is stressed between exposures.**
  - **Movement of object appears as interference fringes.**
    - **Light & dark bands across image.**

Holographic Interferometery

- **Real time holographic interferometry.**
  - Standard hologram of image made.
  - Reconstruct image on top of object.
  - Stress object & interference fringes appear.

Double Exposure Interference Holography

- **Method is as follows**
  - **Set up as for standard holography.**
  - **Take one exposure of object.**
  - **Stress object.**
  - **Take second exposure.**
  - **Develop hologram.**
  - **Reconstruct images.**
- **Two images are on top of each other.**

Analysis

- **From each point on two images, light will have the displacements.**

$$E_1 = E_o \cos\left(kx - \omega t\right)$$

$$E_2 = E_o \cos\left(k\left(x + \Delta x\right) - \omega t\right)$$

- **Δx is movement of that point when object was stressed.**

$$E = E_1 + E_2 = E_o \left\{ \cos\left(kx - \omega t\right) + \cos\left(k\left(x + \Delta x\right) - \omega t\right) \right.$$

$$= E_o \cos\left(k\left(\frac{x + \Delta x}{2}\right) - \omega t \right) \cos\left(\frac{k \Delta x}{2}\right)$$

Analysis

- **Intensity varies as**

$$\cos^2\left(\frac{k\Delta x}{2}\right)$$

- **Intensity shows how much (Δx) object has moved.**